Signal reconstruction and regulators

This manual page describes how the signal is reconstructed and the network constraints (often called regulators) are applied. Network vectors are indicated with the bold font. For example, **w[i]** is the vector representing 0-phase pixel data from all detectors at the time-frequency index i, which is always omitted below.

- the wave is parametrized in the frame (**Fp,Fx**) where the noise scaled antenna pattern vectors **Fp** and **Fx** are defined by the selection of the coordinate frame in the wave plane

(1a) **s** = **Fp***h + e***Fx***H
(1b) **S** = -**Fp***H + e***Fx***h

Here (**s,S**) are the 0^0 and -90^0 responses, (h,H) are the GW quadrature amplitudes (0,90)-degrees phase, e is the ellipticity. For this convention the sign of the quadruple product ([**s x S**],[**Fp x Fx**]) is defined by e.

(2a) $(\mathbf{s*Fp})(\mathbf{S*Fx})-(\mathbf{S*Fp})(\mathbf{s*Fx}) = e*(h^2+H^2)[(|\mathbf{Fp}||\mathbf{Fx}|)^2-(\mathbf{Fp*Fx})^2]$

- the cWB frame is (**Gp,Gx**)

- the DPF frame (f+,fx) is defined as

(3a) **f+** = **Gp***c[d] + **Gx***s[d]; **Gp** = **Fp***c[p] - **Fx***s[p]
(3b) **fx** = **Gx***c[d] - **Gp***s[d]; **Gx** = **Fx***c[p] + **Fp***s[p]

where d is the DPF angle and p is the polarization angle, which defines the conversion between

(**Gp,Gx**)<->(**f+,fx**) and (**Gp,Gx**)<->(**Fp,Fx**)

frames respectively. The conversion between (**f+,fx**) and (**Fp,Fx**) is defined as

(4a) **f+** = **Fp***c[-p+d] + **Fx***s[-p+d] = **Fp***c + **Fx***s
(4b) **fx** = **Fx***c[-p+d] - **Fp***s[-p+d] = **Fx***c - **Fp***s

(4c) **Fp** = **f+***c[-p+d] - **fx***s[-p+d] = **f+***c - **fx***s
(4d) **Fx** = **fx***c[-p+d]+ **f+***s[-p+d] = **fx***c + **f+***s

In the second set of equations and below we drop the sin/cos argument -p+d

- The GW responses in the (**f+,fx**) frame are

$$(5a)\ \mathbf{s} = \mathbf{f+}*(\ h*c+e*H*s) + \mathbf{fx}*(-h*s+e*H*c)$$
$$(5b)\ \mathbf{S} = \mathbf{f+}*(-H*c+e*h*s) + \mathbf{fx}*(\ H*s+e*h*c)$$

$$(5c)\ (\mathbf{s*f+})^2+(\mathbf{S*f+})^2 = |\mathbf{f+}|^4\,(h^2+H^2)\ (c^2+e^2s^2)$$
$$(5d)\ (\mathbf{s*fx})^2+(\mathbf{S*fx})^2 = |\mathbf{fx}|^4\,(h^2+H^2)\ (s^2+e^2c^2)$$

- The standard likelihood analysis reconstructs signal as projections of the data vectors (**w,W**) on the network plane defined by the vectors **f+** and **fx**.

$$(6a)\ wp = (\mathbf{w*f+})/|\mathbf{f+}|^2 \sim\ \ h*c+e*H*s$$
$$(6b)\ wx = (\mathbf{w*fx})/|\mathbf{fx}|^2 \sim -h*s+e*H*c$$

$$(6c)\ Wp = (\mathbf{W*f+})/|\mathbf{f+}|^2 \sim -H*c+e*h*s$$
$$(6d)\ Wx = (\mathbf{W*fx})/|\mathbf{fx}|^2 \sim\ \ H*s+e*h*c$$

- Therefore the GW responses are reconstructed as

$$(7a)\ \mathbf{s} = \mathbf{f+}*(\mathbf{w*f+})/|\mathbf{f+}|^2 + \mathbf{fx}*(\mathbf{w*fx})/|\mathbf{fx}|^2$$
$$(7b)\ \mathbf{S} = \mathbf{f+}*(\mathbf{W*f+})/|\mathbf{f+}|^2 + \mathbf{fx}*(\mathbf{W*fx})/|\mathbf{fx}|^2$$

This solution is trivial for 2-detector networks:

s=w and **S=W**

- To address the problem of trivial solutions, the signal constraints (source model assumptions) and/or regulators SHULD BE applied. Below only the regulators are addressed – e.g. we consider un-modeled search with no assumptions on the source model. Lets define:

$$(8a)\ E+ = (\mathbf{w*f+})^2+(\mathbf{W*f+})^2$$
$$(8b)\ Ex = (\mathbf{w*fx})^2+(\mathbf{W*fx})^2$$

$(8c)\ E\ = (\mathbf{w*w})+(\mathbf{W*W})$ – data energy normalized by noise

$(8d)\ Eh = (h^2+H^2)$ – GW energy normalized by noise

- f+ regulator **delta**: the purpose of this regulator is to diminish signal reconstruction at sky locations with low network sensitivity that unlikely to yield a detectable signal.

(9a) $[E+/E]^{1/2} < |\mathbf{f+}|^2 \, [Eh/E]^{1/2}$
(9b) $f^2 = delta*[E+/E]^{1/2}$ if $delta*[E+/E] > |\mathbf{f+}|^2$
(9c) $f^2 = |\mathbf{f+}|^2$ if $delta*[E+/E] < |\mathbf{f+}|^2$

- fx regulator **gamma**: the purpose of this regulator is to diminish the reconstruction of hx component when |fx|<<|f+|

(10a) $H+ = [(\mathbf{w*f+})^2+(\mathbf{W*f+})^2]/f^4$
(10a) $[Ex/H+]^{1/2} < |\mathbf{fx}|^2 \, [(s^2+e^2c^2)/(c^2+e^2s^2)]^{1/2}$

(10b) $F^2 = gamma*[Ex/H+]^{1/2}$ if $gamma*[Ex/H+] > |\mathbf{fx}|^2$
(10c) $F^2 = |\mathbf{fx}|^2$ if $gamma*[Ex/H+] < |\mathbf{f+}|^2$

- Reconstructed detector responses are

(11a) $\mathbf{s} = \mathbf{f+}*(\mathbf{w*f+})/\mathbf{f}^2 + \mathbf{fx}*(\mathbf{w*fx})/F^2$
(11b) $\mathbf{S} = \mathbf{f+}*(\mathbf{W*f+})/\mathbf{f}^2 + \mathbf{fx}*(\mathbf{W*fx})/F^2$

Given reconstructed responses the signal packets are constructed as described in the wavelet packet manual.