## What are wavelet packets?

Packets are superposition of the wavelet basis functions v[i]: P(t)=Sim_i{c[i]*v[i]} where c[i] are coefficients defined by the wavelet filters. A Wavelet Packet Decomposition (WPD) is obtained by applying the wavelet decomposition steps to the data in the wavelet domain - This is what has been used by coherent waveburst for the analysis of the initial LIGO data. The purpose of such analysis was a uniform tiling of the time-frequency plane.

## The WDM transform already produces a uniform tiling of the TF plane - why WDM packets?

The packet construction presented here has entirely different purpose of solving a specific problem in the time-frequency analysis: A time domain data (x(t),X(t)) from a single detector is converted to the multi-resolution time-frequency data (**w,W**) with the WDM frame (**v,V**), where the low/upper case letters denote 0/90-degrees phase data. The collection of the WDM basis functions (**v,V**) includes several TF-resolutions. Given a TF cluster, the reconstruction targets to obtain the time-domain representation of the cluster (y(t),Y(t)) from (**w,W**). Such time domain representation is constructed as superposition of WDM basis functions v(t) and V(t) - e.g. WDM packet.

## Construction of WDM packets.

Unlike for WDP where the packet coefficients c[i] are fixed, the construction of WDM packets is dependent on data. The procedure is as follows:

- lets define a scalar product (**w|W**)=Sum_i{w[i]*W[i]} and
  norm |**w**|^2=Sum_i{w[i]*w[i]} where w[i]/W[i] are 0/90-degrees phase WDM amplitudes at the TF index i.

- find cos (denoted as c) and sin (denoted as s) for
  transformation of the cluster data vectors (w,W) so the scalar product (**w'|W'**)=0

  (1a) **w'** = **w***c+**W***s ;  s ~ (**w|W**)
  (1b) **W'** = **W***c-**w***s ;  c ~ |**w**|^2-|**W**|^2
  (1c) a = |**w'**|, A = |**W'**| - packet amplitudes
  (1d) u = **w'**/a, U = **W'**/A - packet unity vector

- Define WDM 0/90-degrees phase packets

  (2a) p(t)=Sum_i{u[i]*v[i](t)+U[i]*V[i](t)}=uv(t)+UV(t)
  (2b) P(t)=Sum_i{u[i]*V[i](t)-U[i]*v[i](t)}=uV(t)-Uv(t)

  It should be noted that (p|P)=0 and the sum is taken over pixels in the WDM cluster

- the packet functions satisfy the following identities

(3a) (x|uv) =  a*c,  (x|Uv) = -A*s
(3b) (x|uV) =  a*s,  (x|UV) =  A*c
(3c) (X|uv) = -a*s,  (X|Uv) = -A*c
(3d) (X|uV) =  a*c,  (X|UV) = -A*s

and therefore

(4a) (x|p) = (a+A)*c
(4b) (x|P) = (a+A)*s
(4c) (p|P) = 0 - orthogonal

- given a cluster (**w,W**) the reconstructed time series y(t) is

(5a)  y(t) = (a+A)*c*p(t)/|p|^2 + (a+A)*s*P(t)/|P|^2
(5b) (y|p) = (a+A)*c
(5c) (y|P) = (a+A)*s
here |p|^2 and |P|^2 are the packet norms that are
defined with the WDM cross-talk coefficients:
(v[i]|v[j]), (v[i]|V[j]), (V[i]|V[j])