Detection and reconstruction of burst signals with networks of gravitational wave detectors

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LIGO Scientific Collaboration

Outline

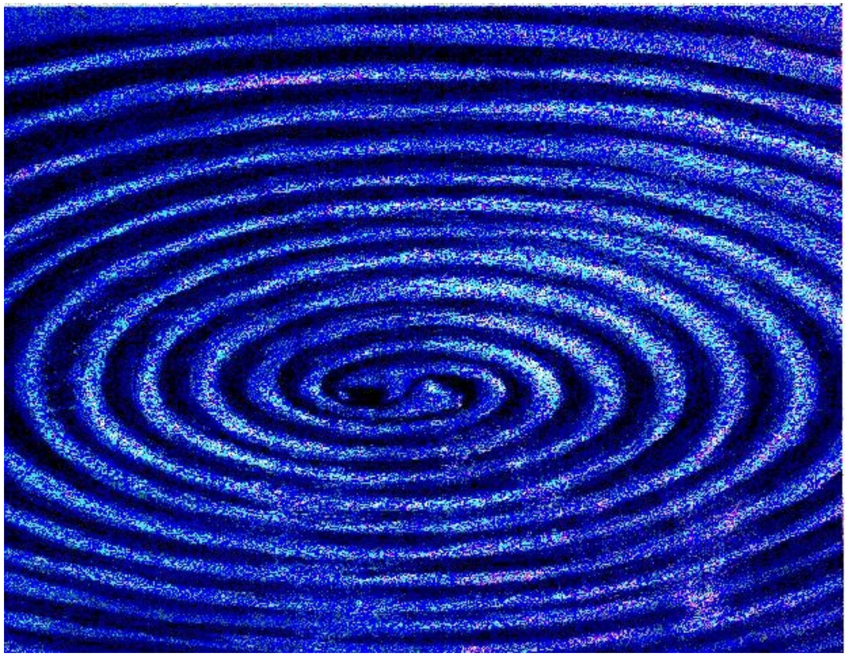
- Gravitational Waves
  - bursts
- Gravitational wave detectors
  - Detector response
  - Networks of GW detectors
- Detection of GW signals
  - Coherent network analysis
  - Constraint likelihood
  - Consistency tests for burst events
- Reconstruction of GW signals
- Summary

gravitational waves

- time dependent gravitational fields come from the acceleration of masses and propagate away from their sources as a space-time warpage at the speed of light

- In the weak-field limit, linearize the equation in "transverse-traceless gauge"

$$\nabla^2 h - \frac{\partial^2 h}{c^2 \partial t^2} = 16\pi \frac{G_N}{c^4} T$$



gravitational radiation
binary inspiral of compact objects

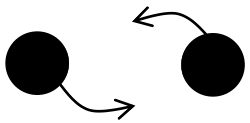
where $h_{\mu\nu}$ is a small perturbation of the space-time metric

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

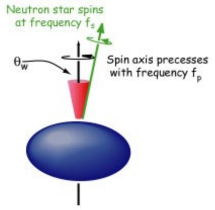
Gravitational waves

- Perturbation of space-time metric predicted by GR

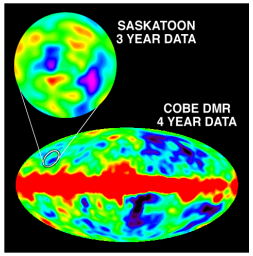
- Compact binary inspiral: **“chirps”**
  - neutron stars / black holes



- Pulsars in our galaxy: **“periodic”**
  - GW from observed neutron stars



- Cosmological/astrophysical signals: **“stochastic”**
  - Early universe (like CMBR) or unresolved sources



- Supernovae / GRBs/ BH mergers/…: **“bursts”**
  - triggered – coincidence with GRB/neutrino detectors
  - un-triggered – coincidence of GW detectors

Detectors

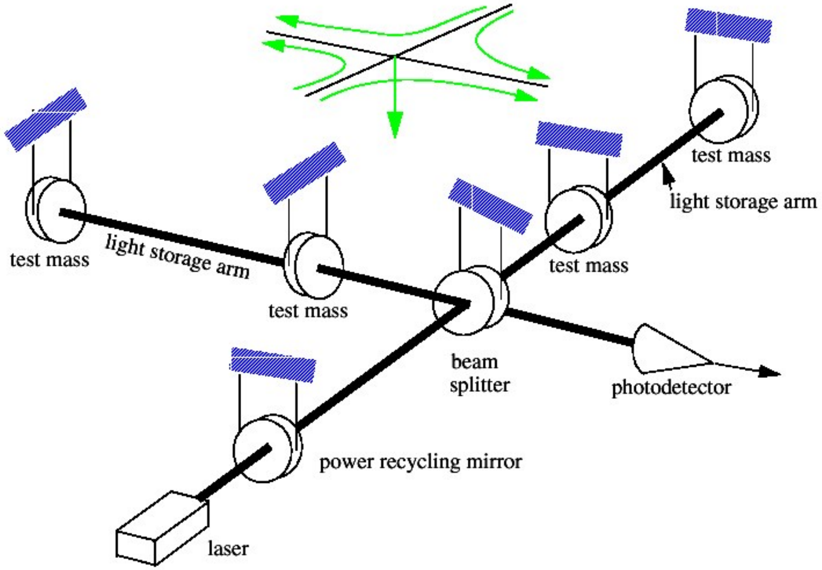
Bars

narrowband (~1Hz)
recent improvements (~10Hz)

ALLEGRO, AURIGA, EXPLORER, NAUTILUS, NIOBE, …

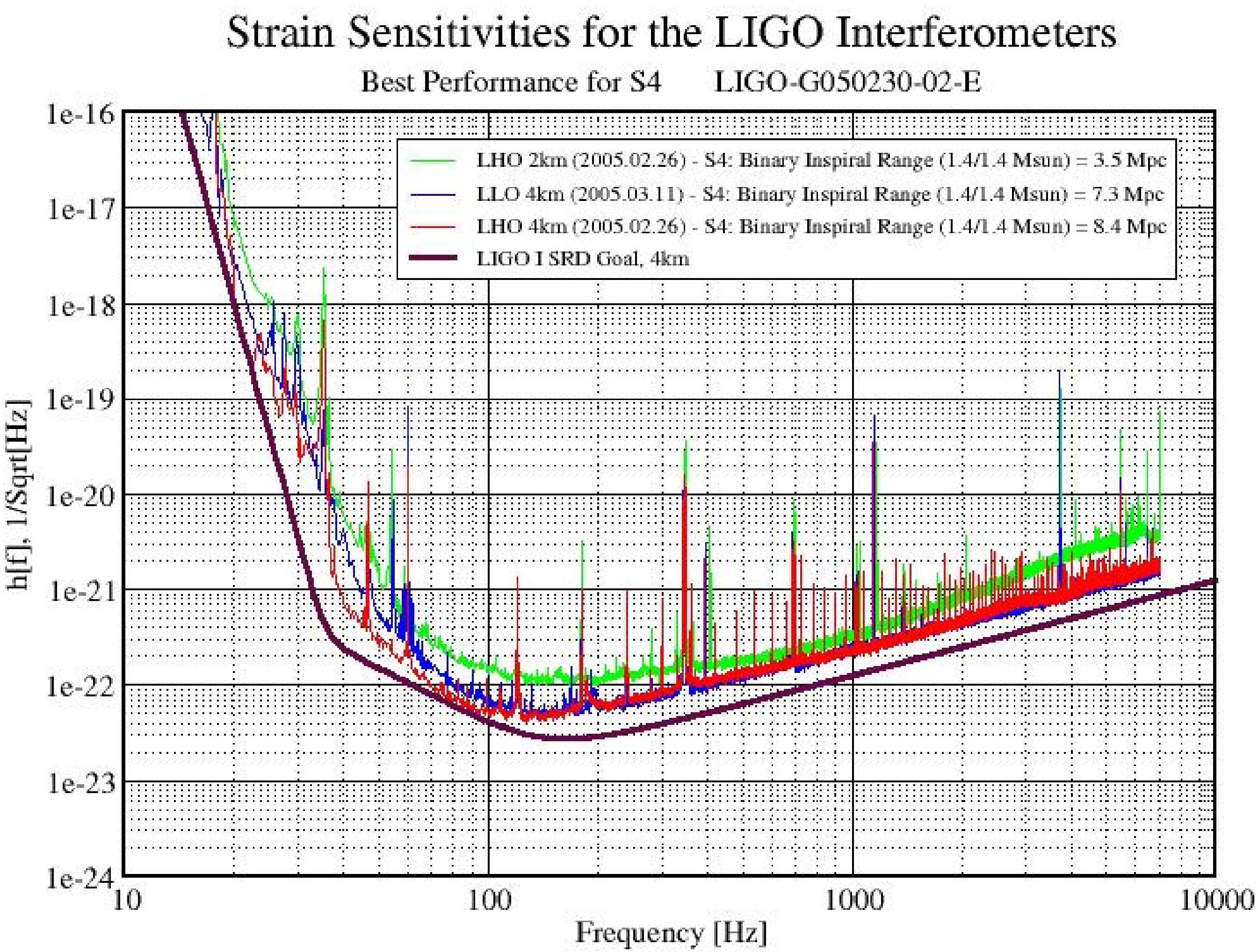
Interferometers

wideband (~10000 Hz)



LIGO, VIRGO, GEO, TAMA, AIGO, …

LIGO Sensitivity



LIGO achieved design sensitivity in S5 run which is complete

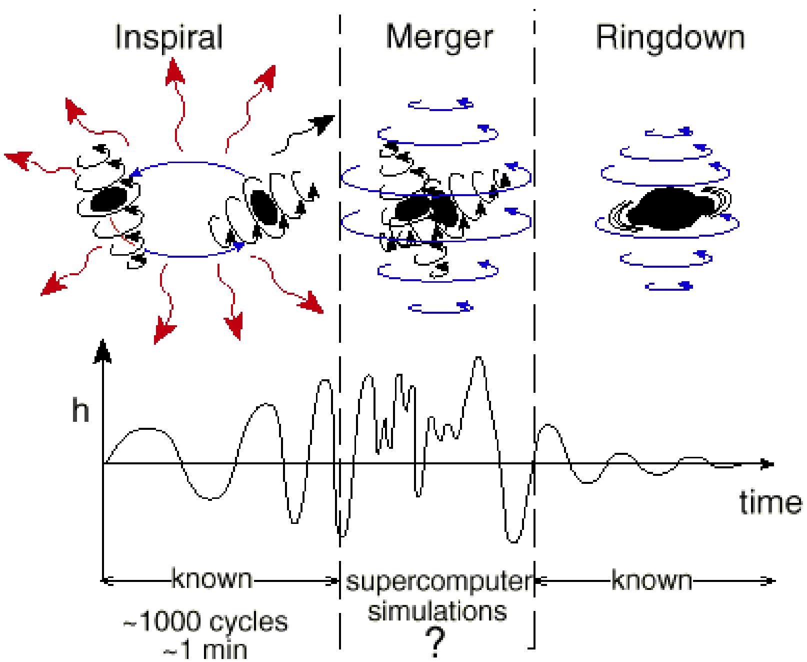
Gravitational Waves from Bursts Sources

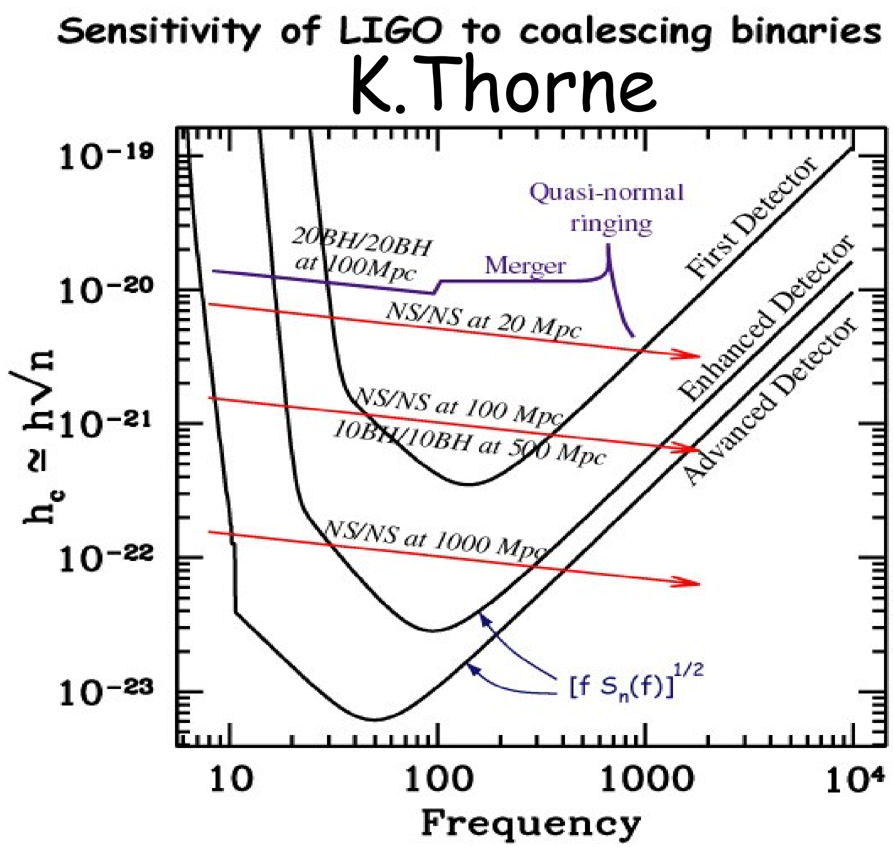
- Any short transient of gravitational radiation (< few sec).
- Astrophysically motivated
  - Un-modeled signals -- Gamma Ray Bursts, …
  - Poorly modeled -- supernova, inspiral mergers,..
  - Modeled – cosmic string cusps
- In most cases matched filters will not work
- Characterize un-modeled bursts by
  - characteristic frequency *fc*
  - duration (δt) & bandwidth (δf) & TF volume (δt X δf)
  - strain amplitude h_{rss}

$$h^2{}_{rss} = \int_{-\infty}^{+\infty} \left[h_+^2(t) + h_\times^2(t) \right] dt$$

Inspiral Mergers

Compact binary mergers





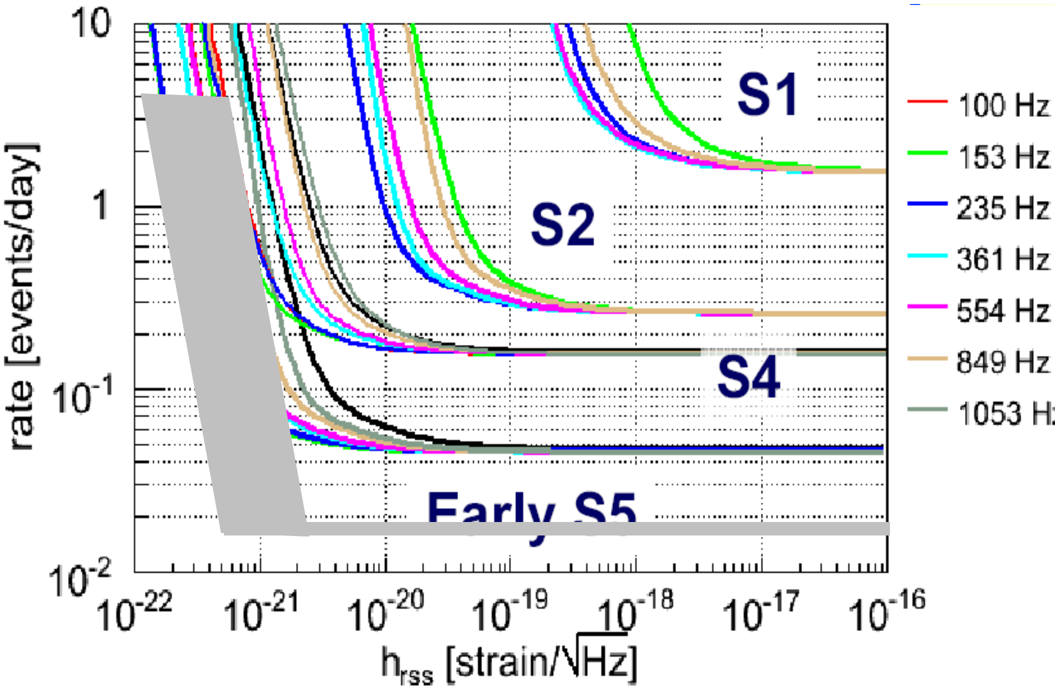
- massive BH-BH objects can be detected via merger and ring-down
- One of the most promising source to be detected with LIGO
- Recent progress in NR will help to extract information about BH-BH dynamic when mergers are detected.

LIGO burst searches

- use WaveBurst algorithm
  (Klimenko et al, CQG 21, S181 (2004))
  to generate triggers reconstructed in wavelet (time-frequency) domain

- use CorrPower algorithm
  (Cadonati et al, CQG 21, S181 (2004))
  for consistency test of triggers

Abbot et al, PRD **69**, 102001 (2004)
Abbot et al, PRD **72**, 062001 (2005)



- Set rate vs strength upper limit on generic GW bursts
- S2: set limit on rate <0.26 events/day at 90% conf. level
- S4: significant improvement in sensitivity (x10)
- S5: significant increase of life time (x10), analysis in progress

Objectives of Coherent Burst Analysis

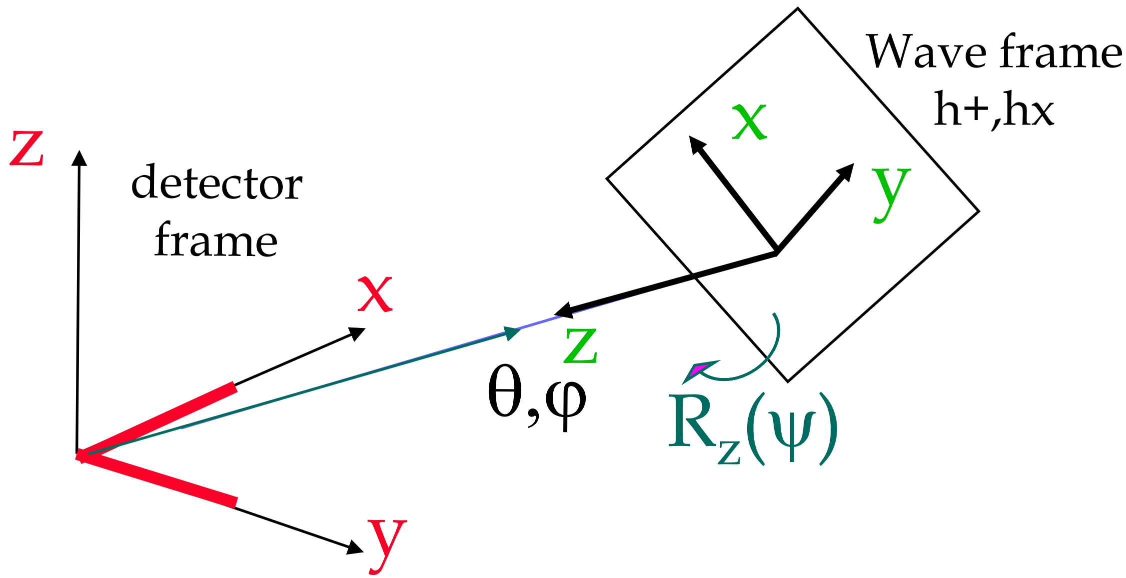
- Combine measurements from several detectors
  - confident detection, elimination of instrumental/environmental artifacts
  - reconstruction of source coordinates
  - reconstruction of GW waveforms
- Detection & reconstruction methods should handle
  - arbitrary number of co-aligned and misaligned detectors
  - variability of the detector responses as function of source coordinates
  - differences in the strain sensitivity of detectors
- Extraction of source parameters
  - confront measured waveforms with source models
- For burst searches matched filters do not work
  - need robust model independent detection algorithms

Coherent network analysis

Combine data, not triggers

- Guersel,Tinto, PRD 40 v12,1989
  - reconstruction of GW signal for a network of three misaligned detectors
- Likelihood analysis: Flanagan, Hughes, PRD57 4577 (1998)
  - likelihood analysis for a network of misaligned detectors
- Two detector paradox: Mohanty et al, CQG 21 S1831 (2004)
  - state a problem within likelihood analysis
- Constraint likelihood: Klimenko et al, PRD 72, 122002 (2005)
  - address problem of ill-conditioned network response matrix
  - first introduction of likelihood constraints/regulators
- Penalized likelihood: Mohanty et al, CQG 23 4799 (2006).
  - likelihood regulator based on signal variability
- Maximum entropy: Summerscales at al, to be published
  - likelihood regulator based on maximum entropy
- Rank deficiency of network matrix: Rakhmanov, CQG 23 S673 (2006)
  - likelihood based in Tickhonov regularization
- GW signal consistency: Chatterji et al, PRD 74 082005(2006)
  - address problem of discrimination of instrumental/environmental bursts
- Several Amaldi7 presentations and posters by I.Yakushin, S. Chatterj, A.Searle and S.Klimenko

GW signal and detector response



- **Direction to the source θ,φ and polarization angle Ψ define relative orientation of the detector and wave frames.**
- **two GW polarizations:** $\vec{h} = \left(h_+(t), h_\times(t)\right)$
- **Antenna patterns:** $\vec{F} = \left(F_+(\theta,\varphi), F_\times(\theta,\varphi)\right)$
- **Detector response:** $\xi = F_+ h_+ + F_\times h_\times = \vec{F} \cdot \vec{h}$

Likelihood

- **Likelihood for Gaussian noise with variance $\sigma^2{}_k$ and GW waveforms h_+, h_x : $x_k[i]$ – detector output, F_k – antenna patterns**

$$L = \sum_i \sum_k \frac{1}{\sigma_k^2}\left[x_k^2[i] - \left(x_k[i] - \xi_k[i]\right)^2\right]$$

detector response - $\xi_k = h_+ F_{+k} + h_\times F_{\times k}$

- **Find solutions by variation of *L* over un-known functions h_+, h_x (Flanagan & Hughes, PRD 57 4577 (1998))**

- **"Matched filter" search in the full parameter space**
  - **good for un-modeled burst searches, but…**
  - **number of free parameters is comparable to the number of data samples**
  - **need to reduce the parameter space → constraints & regulators (Klimenko et al , PRD 72, 122002, 2005)**

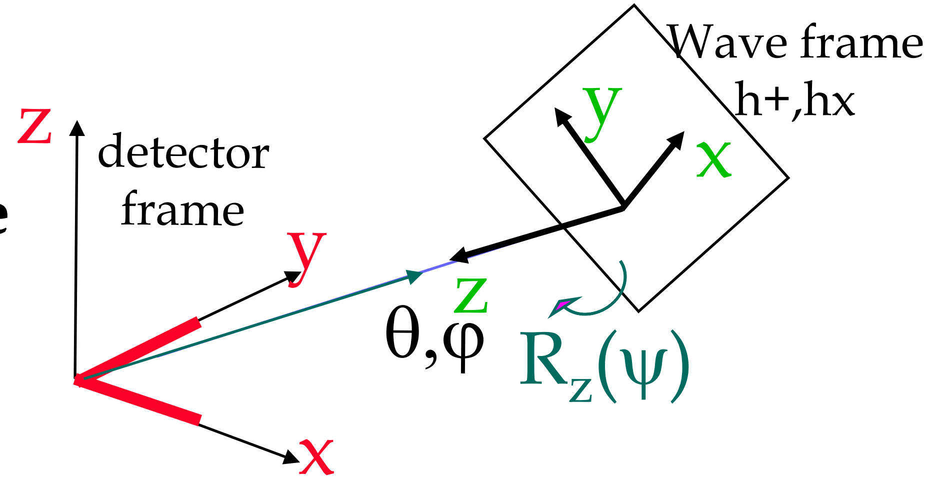
Network response matrix

$$\vec{X} = \left\{ \frac{x_k}{\sigma_k} \right\}, \quad \vec{F}_{+,\times} = \left\{ \frac{F_{+,\times k}(\Psi_{DPF})}{\sigma_k} \right\}$$

- **Dominant Polarization Frame**

$$\vec{F}_+ \cdot \vec{F}_\times = 0$$

(all observables are $R_Z(\Psi)$ invariant)



- **solution for GW waveforms satisfies the equation**

$$\begin{bmatrix} \vec{X} \cdot \vec{F}_+ \\ \vec{X} \cdot \vec{F}_\times \end{bmatrix} = \begin{bmatrix} |F_+|^2 & 0 \\ 0 & |F_\times|^2 \end{bmatrix} \begin{bmatrix} h_+ \\ h_\times \end{bmatrix} = g \begin{bmatrix} 1 & 0 \\ 0 & \varepsilon \end{bmatrix} \begin{bmatrix} h_+ \\ h_\times \end{bmatrix}$$

- ***g*** **– network sensitivity factor**
- ***ε*** **– network alignment factor**

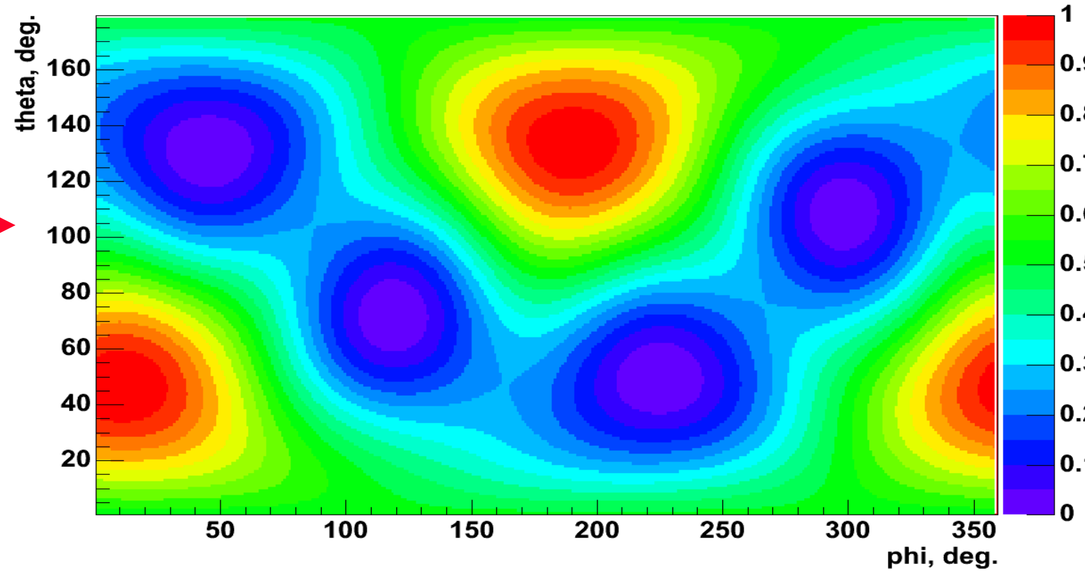
network response matrix

(PRD 72, 122002, 2005)

Detector Antenna Patterns

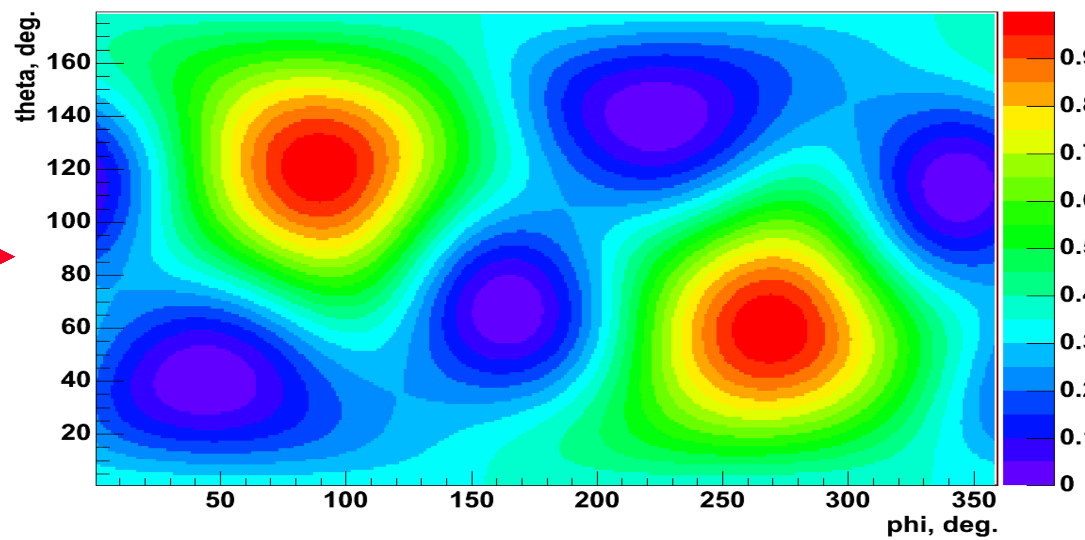
$$A = \frac{1}{2}\left(F_{+} + iF_{x}\right)$$

- $|A|^2$ for L1 →



- Several misaligned detectors increase coverage of the sky

- $|A|^2$ for Virgo →



Detection of two GW components

- h_1 & h_2 - solutions for GW polarizations in the DP frame
- For aligned detectors ε=0 for any θ and ϕ
- For misaligned detectors ε can be <<1 for significant area in the sky

- total network SNR

$$L \approx g\left(\left\langle h_1^2 \right\rangle + \varepsilon \left\langle h_2^2 \right\rangle\right) = SNR_{tot}$$

$\left\langle h_1^2 \right\rangle, \left\langle h_2^2 \right\rangle$ -sum-square energies of GW components

- if ε=0 only component h_1 can be measured
- Even for networks with several misaligned detectors the measurement of the second component not always possible

Network alignment factor

$$\varepsilon = p - |q| \,/\, p + |q|$$

For aligned network ε=0

ε shows relative sensitivity to two GW components

$$L \propto \left(\langle h_1^2 \rangle + \varepsilon \langle h_2^2 \rangle \right)$$

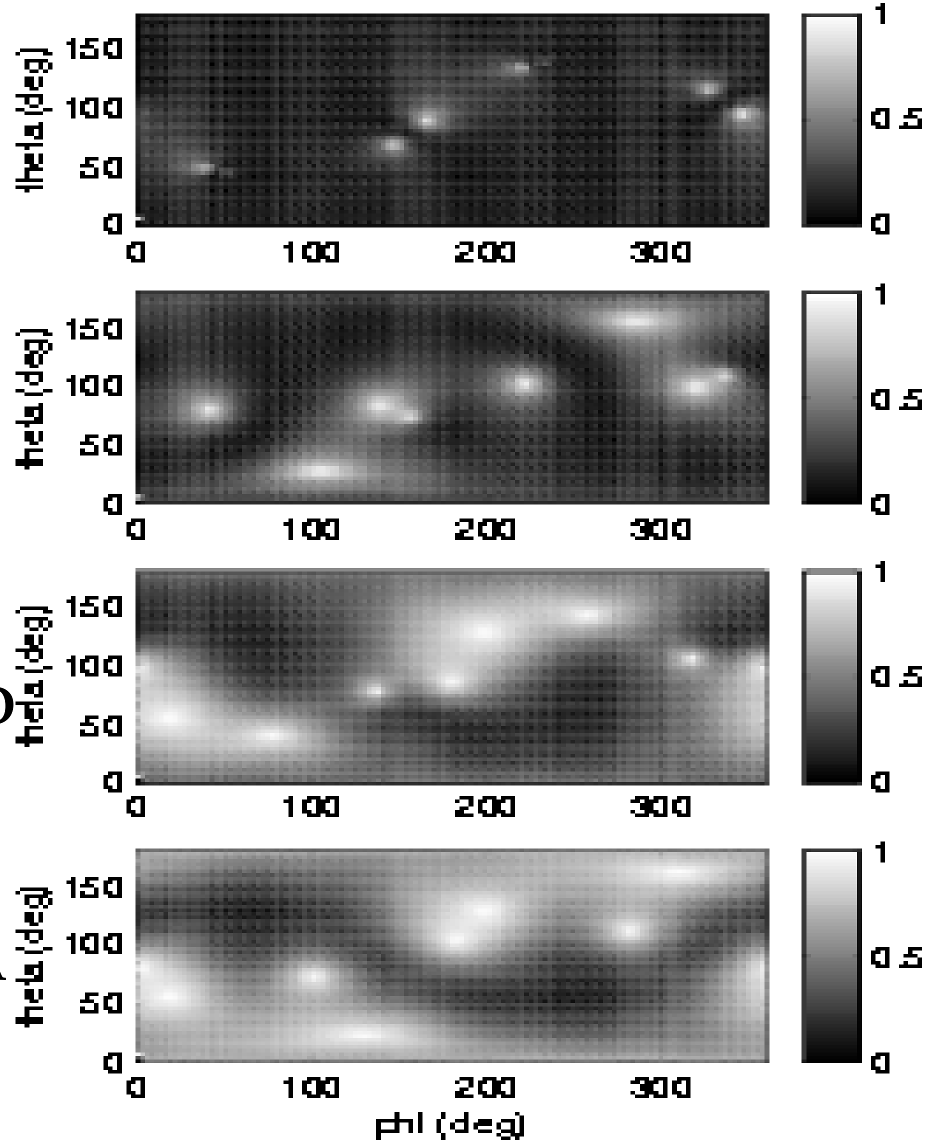
to be detected with the same SNR h_2 should be 1/ε times stronger then h_1

H1-L1

+GEO

+VIRGO

+TAMA



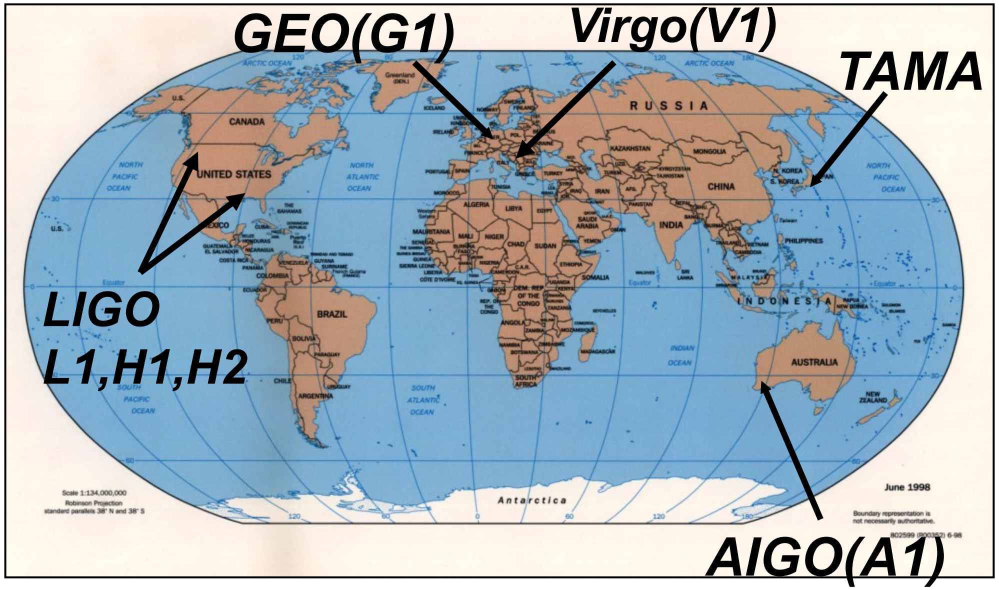
Global Network of GW detectors

$$|\vec{F}_{+,\times}|^2 = \sum_k \frac{F^2_{+,\times k}}{\sigma_k^2}$$

$$g = |\vec{F}_+|^2, \quad \varepsilon = \frac{|\vec{F}_\times|^2}{|\vec{F}_+|^2}$$

detector: L1:H1:H2:G1:V1:A1

σ^2_k: 1 : 1 : 4 : 10 : 1 : 1



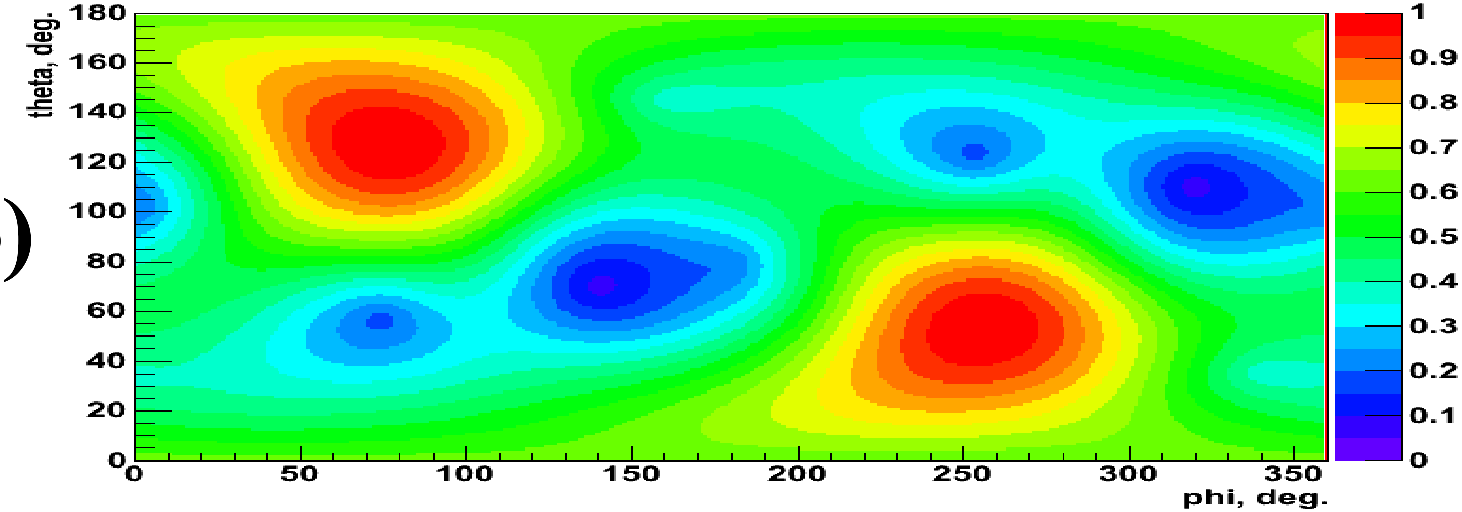
g_a and ε_a are averaged over the sky

network	g_a	ε_a,%	θ,ϕ	rejection of glitches
single IFO	1	0	-	-
H1/H2	1.4	0	-	H1-H2 consistency (correlated noise?)
H1/H2/L1	2.3	2.7	ring	waveform consistency
H1/H2/L1/G1	2.4	4.8	ring-point	waveform consistency
H1/H2/L1/G1/V1	3.1	16.5	ring-point	waveform consistency

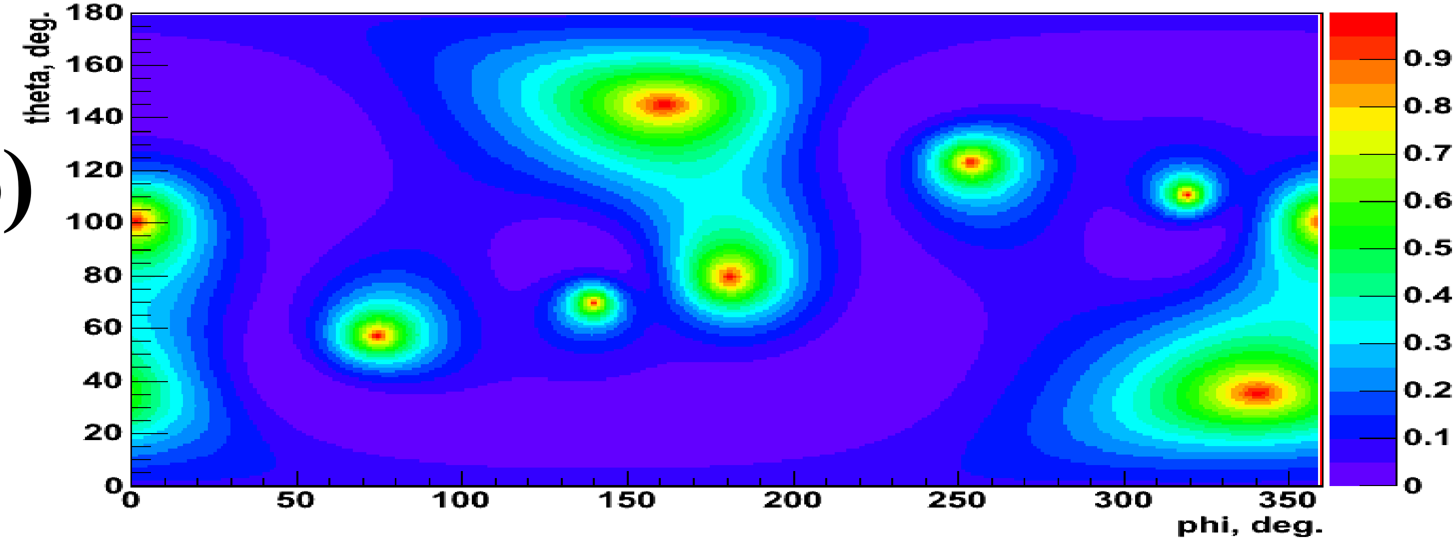
L1/H1/V1 network

- For better reconstruction of waveforms (and source parameters) more coverage on the second polarization is desirable

$g(\theta,\phi)$

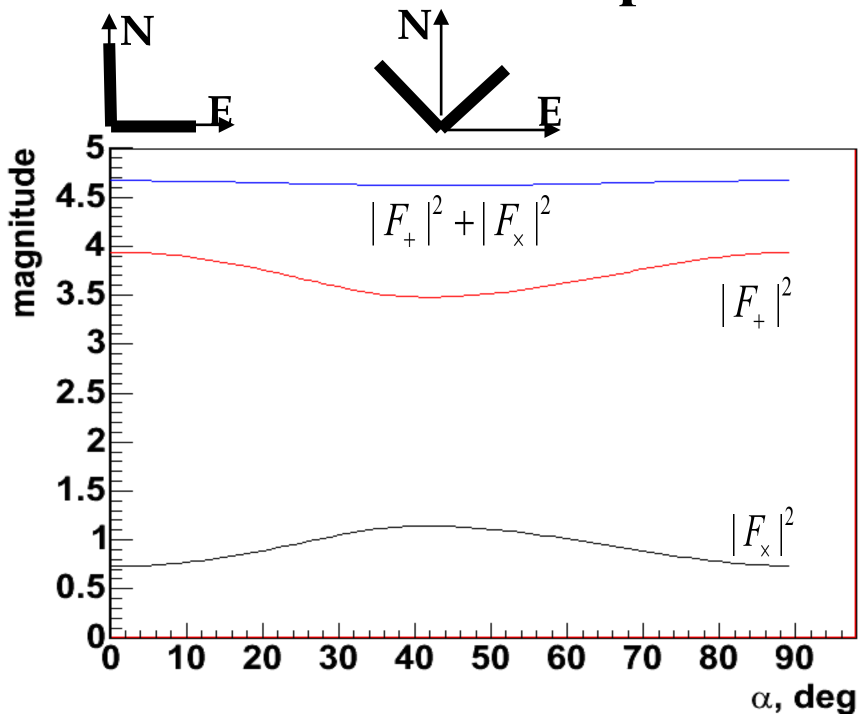


$\varepsilon(\theta,\phi)$

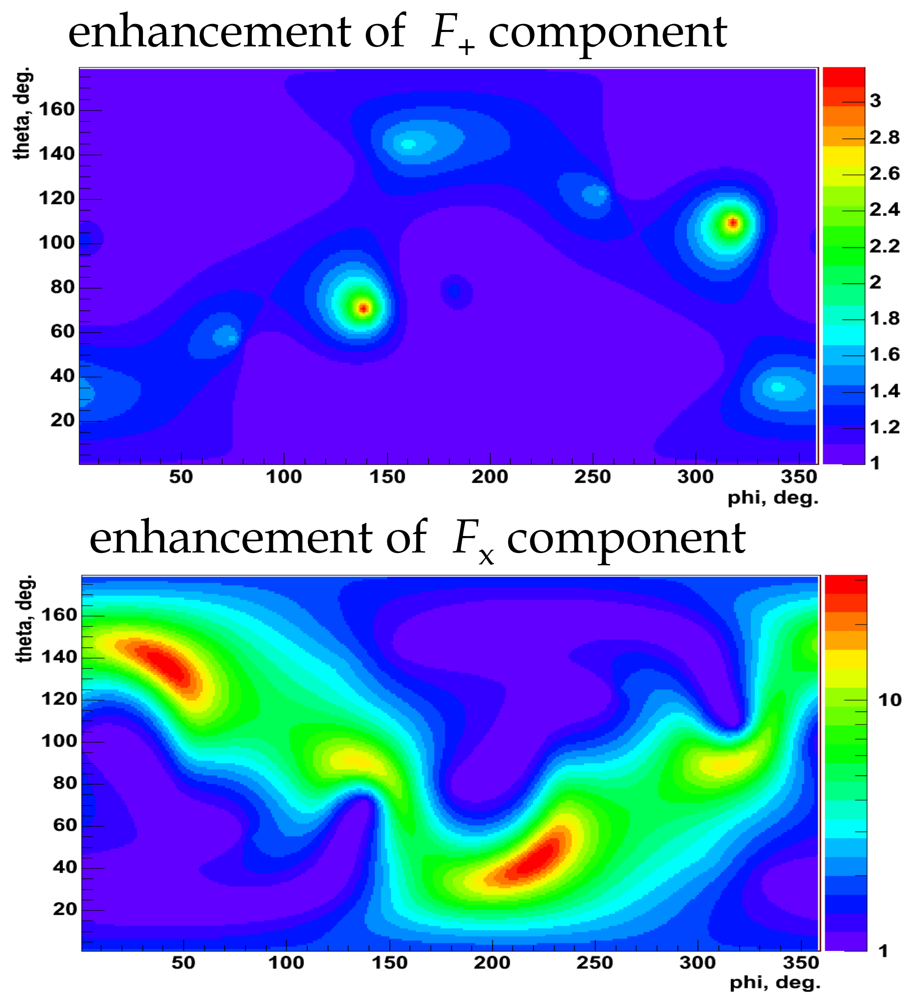


Optimal orientation of future detectors

- **AIGO is almost antipodal to LIGO (lat: 121.4, long: -115.7)**



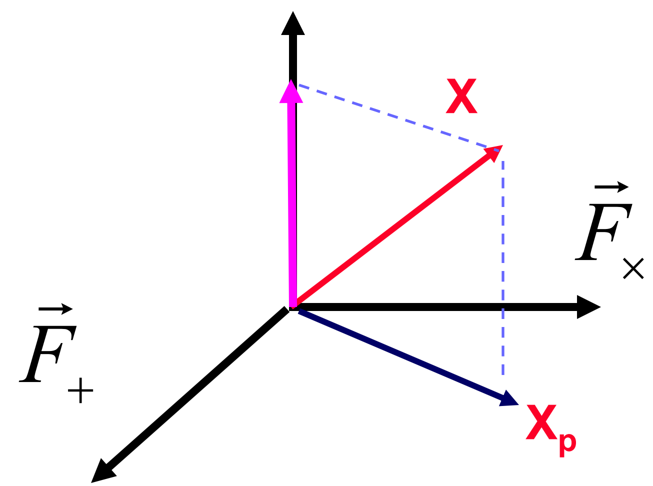
network	g_a	ε_a,%
H1/H2/L1/G1/V1	**3.1**	**16.5**
H1/H2/L1/G1/V1/A1	**3.5**	**33.0**



- **significant improvement in the detection of the second polarization**

Network projections

$$\vec{F}_+ \cdot \vec{F}_\times = 0$$



- **Likelihood ratios**

$$L_+ = \frac{\left(\vec{X}\cdot\vec{F}_+\right)^2}{|F_+|^2} = X^T P_+ X, \quad P_{ij} = \frac{F_{+i}F_{+j}}{|F_+|^2} = \mathrm{e}_{+i}\mathrm{e}_{+j}$$

$$L_\times = \frac{\left(\vec{X}\cdot\vec{F}_\times\right)^2}{|F_\times|^2} = X^T P_\times X, \quad P_{ij} = \frac{F_{\times i}F_{\times j}}{|F_\times|^2} = \mathrm{e}_{\times i}\mathrm{e}_{\times j}$$

- **regulators are introduced to construct P_x when $|F_x| \to 0$**
  hard, soft, Tikhonov, etc..

Two detector paradox

Mohanty et al, CQG 21 S1831 (2004)

- **for simplicity assume unit noise variance**
- **aligned detectors (identical detector responses ξ):**

$$L=\sum_i \xi[i]\left(x_1[i]+x_2[i]-\xi[i]\right) \;\Rightarrow\; \xi=\frac{x_1+x_2}{2}$$

$$L_A=\tfrac{1}{4}\left[\langle x_1,x_1\rangle+\langle x_2,x_2\rangle+2\langle x_1,x_2\rangle\right]$$

power — cross-correlation

  - **If separated → L_A has directional sensitivity (circle on the sky) because correlation term depends on θ and ϕ.**
- **misaligned detectors:**
  - **solution for GW waveform:** $\xi_1=x_1,\quad \xi_2=x_2$

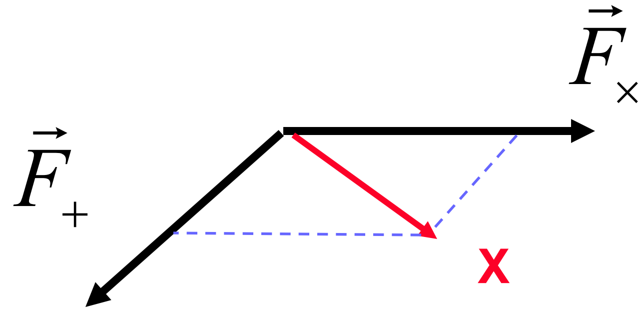
$$L_M=\tfrac{1}{2}\left[\langle x_1,x_1\rangle+\langle x_2,x_2\rangle\right]$$

- **Likelihood method does not work for two misaligned detectors**

No directional sensitivity even if detectors are infinitesimally misaligned!

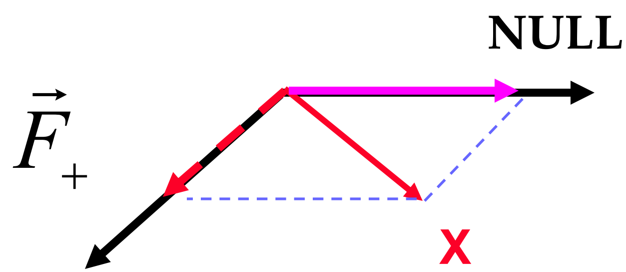
Two detector case

- **Misaligned detectors**
  - no null space
  - ε <<1 for significant fraction of the sky
  - L=const(θ,ϕ)



$$L_+ + L_\times = x_1^2 + x_2^2$$

- **Aligned detectors (H1H2)**
  - $\varepsilon = 0$
  - only one projection P+



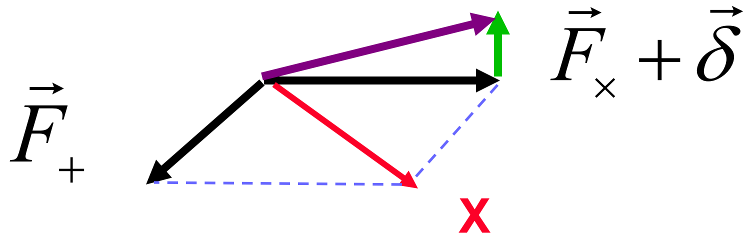
- **The discontinuity between aligned and misaligned cases can be resolved with regulators:** $|F'_\times|^2 = |F_\times|^2 + \delta$

Treatment of regulators

- regulators can not be arbitrary - they should preserve the orthogonality of the network vectors F_+ and F_x . Otherwise the projections P_+ and P_x can not be constructed.
- regulators can be introduced in two (equivalent) ways by adding small non-zero vector δ to F_x

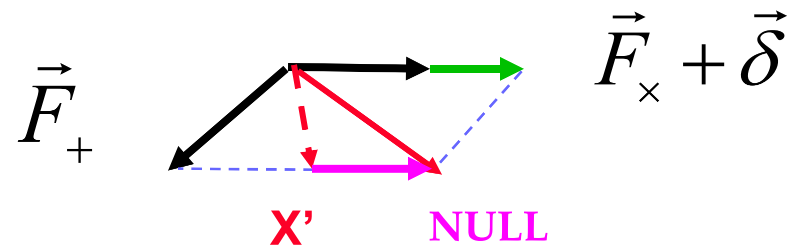
> "dummy detector"

$$\vec{F}'_+ = \{F_{+1}, F_{+2}, 0\}$$
$$\vec{F}'_\times = \{F_{\times 1}, F_{\times 2}, \delta\}$$
$$\vec{X} = \{x_1, x_2, 0\}$$



> split the X-axis

$$|F'_\times|^2 = (|F_\times| + |\delta|)^2$$



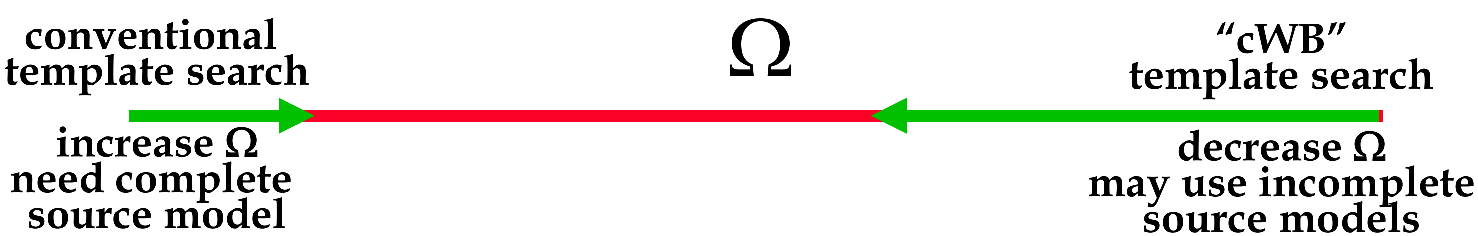
Coherent WaveBurst (cWB)

- **End-to-end multi-detector coherent search**
  - **handle arbitrary number of co-aligned and misaligned detectors**
  - **reconstruction of source coordinates and GW waveforms & detector responses**
  - **use coherent statistics for elimination of instrumental/environmental artifacts**

- **Template search in the full parameter space**

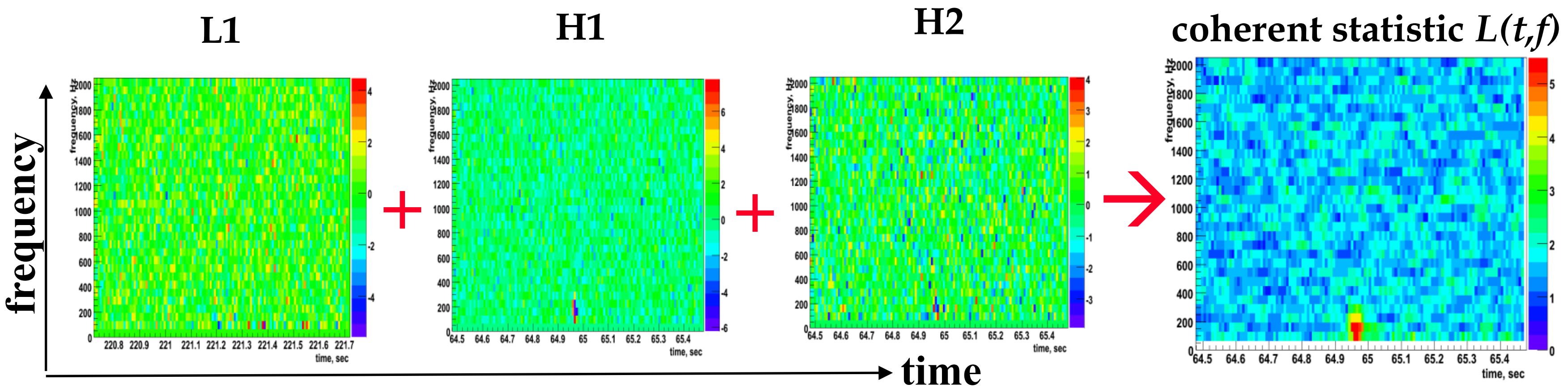
$$L(x\,|\,h_+,h_\times,\Omega) = -\ln\left(\max_s\left(\frac{P(x\,|\,h_+(\Omega),h_\times(\Omega))}{P(x\,|\,0)}\right)\right) \rightarrow \Omega \equiv \{h_+,h_\times\}$$

  - **Find solutions by variation of *L* over un-known functions h_+, h_x**
    (Flanagan & Hughes PRD 57 4577 (1998))
  - **good for un-modeled burst searches, but…**
  - **number of free parameters is too large (~ *DOF*)**
  - **need to reduce the parameter space → constraints & regulators**
    (Klimenko et al , PRD 72, 122002, 2005)



Coherent statistic

- construct coherent statistic for detection
- perform search over ~65000 sky locations
- perform analysis for ~100 time shifts for background estimation

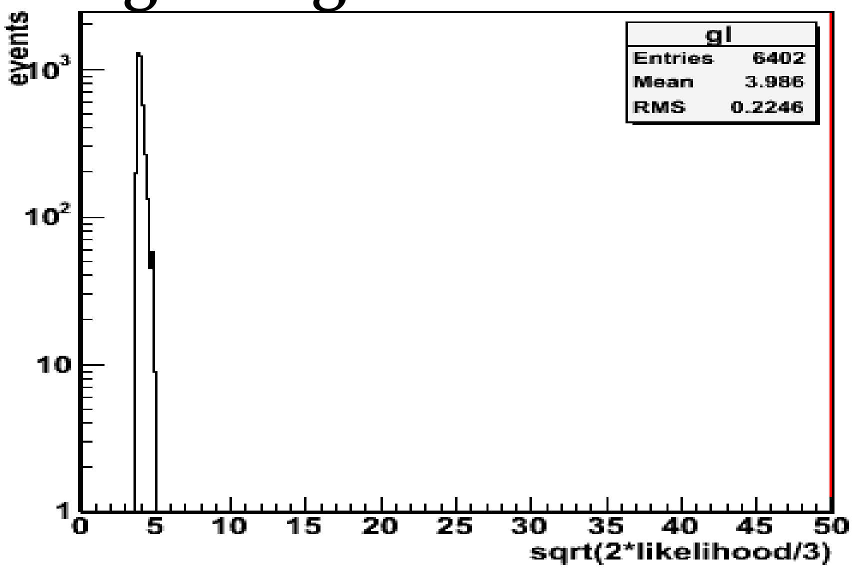


$$L(t,f) = \max_{h_+ h_\times \theta\varphi} \sum_k \frac{x_k^2[t,f] - \left(x_k[t,f] - \xi_k[t,f]\right)^2}{\sigma_k^2(f)}$$

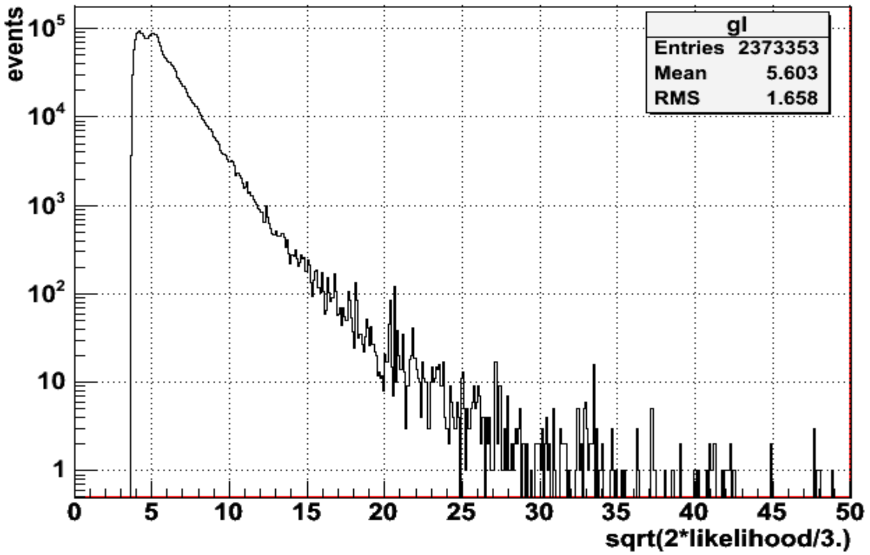
$$\xi_k = h_+ F_{+k} + h_x F_{xk}$$

Consistency test of network triggers

Ligo-Virgo simulated data



S4 data



- Likelihood statistic is designed to separate non-stationary bursts from stationary Gaussian noise
- Real data is dominated by glitches
- The coherent statistics is a powerful tool to reject glitches
- Consistency test for LIGO and LIGO-GEO data based on
  - reconstructed burst energy in individual detectors
  - network correlation

Consistency Test of coincident events

- **Are triggers detected in different detectors consistent?**
- **Pearson's correlation between two detector data streams: r-statistic, Cadonati, CQG 22 S1159 (2005)**
  - **can test a consistency of waveforms in the detectors, works for co-aligned or closely aligned detectors**
  - **effective tool for FA reduction, successfully used in LIGO burst searches**
- **Null stream: Schutz et al, CQG 22 S1321 (2005)**
  - **construct linear combination of data streams where GW signal is cancelled out. Reject triggers if residual is not consistent with the noise**
  - **most straightforward is a null stream for co-aligned detectors: P.Ajith et al, CQG 23 S741-S749 (2006)** $N(t) = x_1(t) - x_2(t+\tau)$
- **Both methods can significantly reduce false alarm, but they mainly work for co-aligned detectors and do not address the problem of GW reconstruction.**

Coherent statistics

- **Likelihood: estimator of network SNR → detection statistic**

$$L = \sum_i \sum_k \frac{1}{\sigma_k^2}\left[x_k^2[i] - \left(x_k[i] - \xi_k[i]\right)^2\right]$$

$$L = E - N$$

detected (signal) energy → L; **total energy** → E; **noise (null) energy** → N

- **Individual statistics L_k, E_k, N_k for each detector are also available**
- **Likelihood matrix**

$$L = \sum_{i,j} \left\langle x_i x_j e_i e_j \right\rangle = E_{i=j} + E_{i \neq j}$$

incoherent → $E_{i=j}$; **coherent** → $E_{i \neq j}$

correlation of misaligned detectors

- **Correlated energy** $E_{coherent} = \sum_{i \neq j} L_{ij}$

- **Pearson's statistic**

$$r_{ij} = \frac{L_{ij}}{\sqrt{L_{ii} L_{jj}}} \rightarrow \frac{\langle x_i x_j \rangle}{\sqrt{\langle x_i^2 \rangle \langle x_j^2 \rangle}}$$

any detectors   two aligned detectors

- **network correlation coefficient**

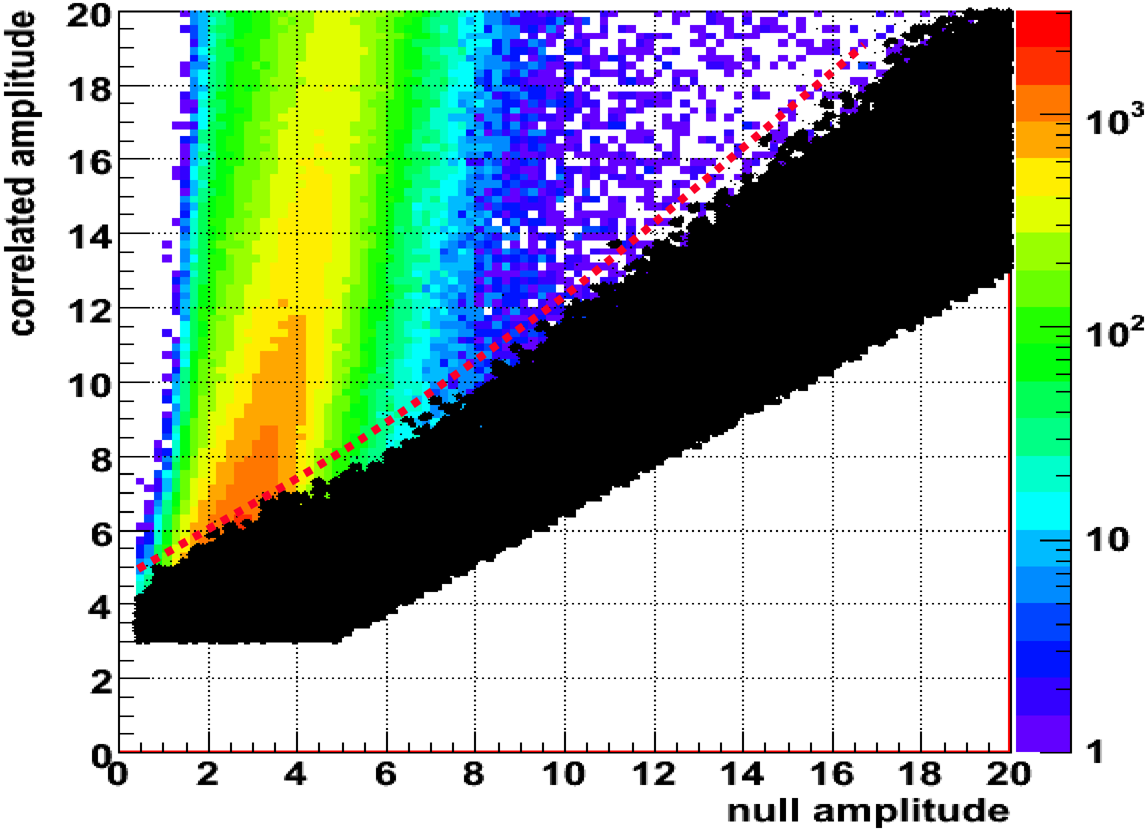
$$C_{net} = \frac{\sum_{i \neq j} L_{ij}}{E - \sum L_{ii}} = \frac{E_{coherent}}{N_{ull} + E_{coherent}}$$

coherent – null energy

- **coherent energy: sum of the off-diagonal elements of L matrix (in PCF)**

$$E_{coherent} = \sum_{i \neq j} L_{ij}$$

- **null energy *null*: energy of the reconstructed detector noise**



In-coherent/Coherent Energy

$$L_+ = \sum_{i,j} x_i x_j P_{ij,+} = E_{+(i=j)} + C_{+(i\neq j)}$$

$$L_\times = \sum_{i,j} x_i x_j P_{ij,\times} = E_{\times(i=j)} + C_{\times(i\neq j)}$$

- **quadratic forms C_+ & C_x depend on time delays between detectors and carry information about θ,ϕ – sensitive to source coordinates**
- **properties of the likelihood quadratic forms**

arbitrary network

$$\mathrm{cov}(L_+ L_\times) = 0$$

$$\mathrm{cov}(C_+ C_\times) = -\sum \mathrm{e}_{+i}^2 \mathrm{e}_{\times i}^2$$

$$\mathrm{cov}(E_+ E_\times) = \sum \mathrm{e}_{+i}^2 \mathrm{e}_{\times i}^2$$

2 detector network

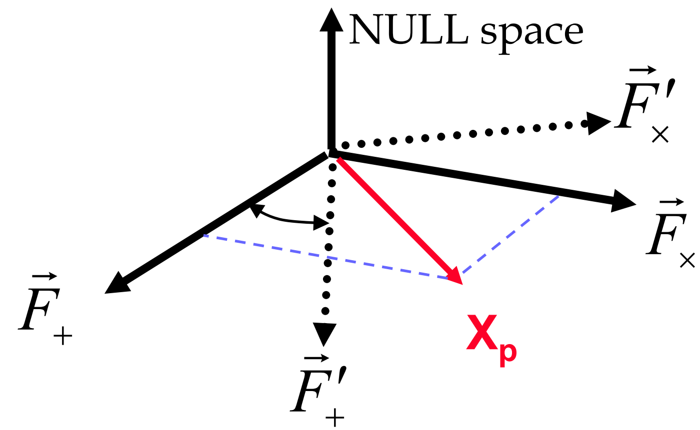
$$C_+ + C_\times = 0$$

$$E_+ + E_\times = \mathrm{x}_1^2 + \mathrm{x}_2^2$$

- **How is the coherent energy defined?**

Principle Component Frame

- **L, null stream and reconstructed waveforms are invariant with respect to rotation in the projection sub-space**

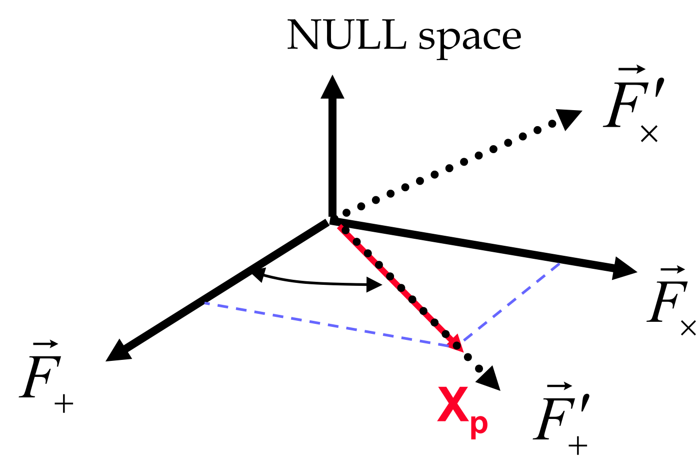


- **But incoherent & coherent terms depend on the selection of the coordinate frame**

- **Define coherent energy in the frame where F′$_+$ is aligned with the projection of X (X_p) (*principle component frame*)**

$$L'_+ = L_+ + L_\times$$

$$L'_\times = 0, \qquad E'_\times = -C'_\times$$



- **coherent/incoherent energies**

$$C = \sum_{i \neq j} x_i x_j e'_{i+} e'_{j+} \qquad E = \sum_i x_i x_i e'_{i+} e'_{i+}$$

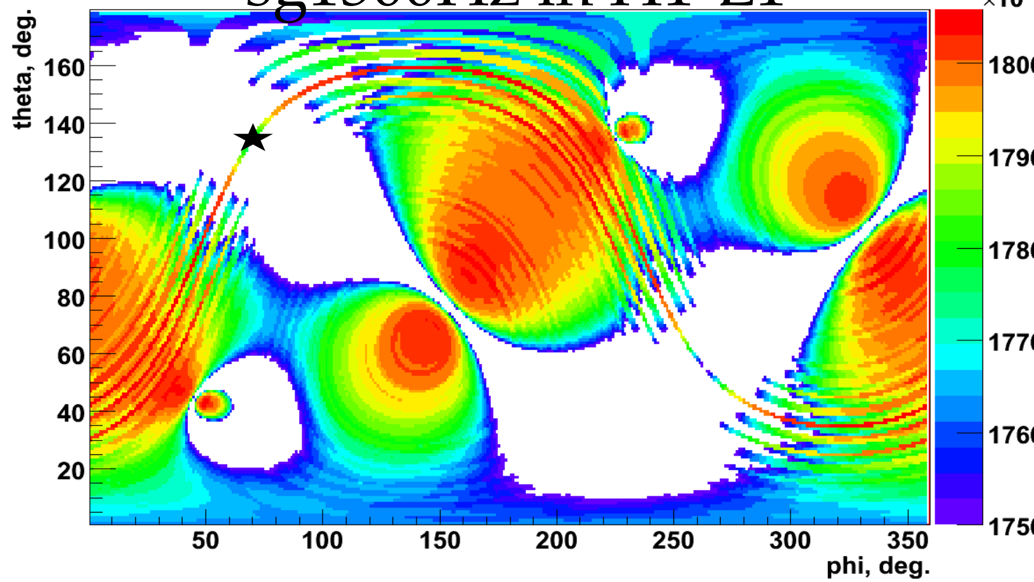
Coordinate reconstruction

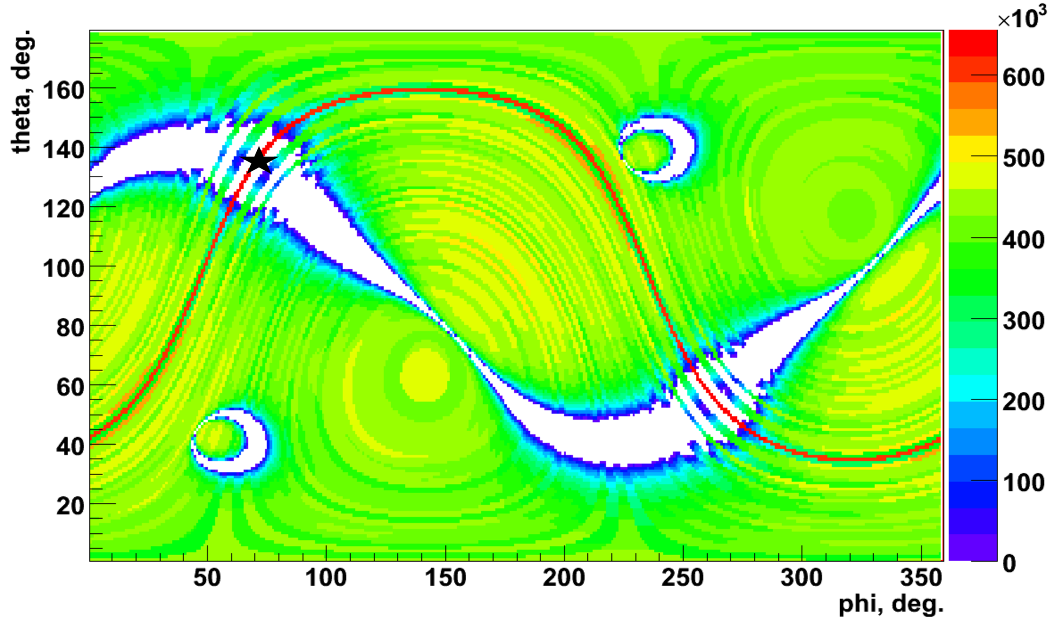
- **What statistic to use?**

- **Likelihood ratio**
  - very dependent on regulators
  - large bias

- **Correlated Energy**
  - sensitive to time delays
  - calculated in PCF
  - works with “right” regulator,
  - little dependence on regulator
  - small bias

sg1300Hz in H1-L1



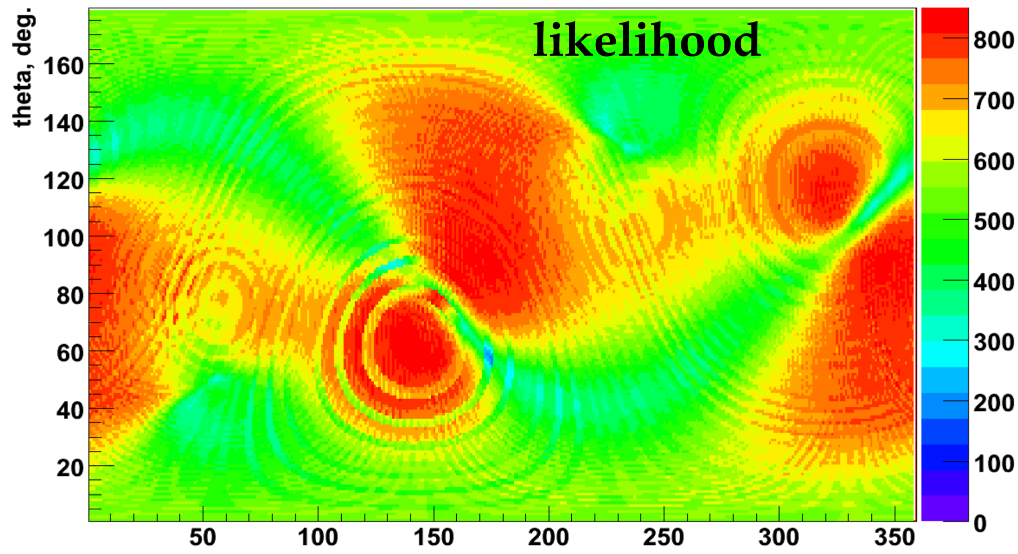


Reconstruction of source coordinates

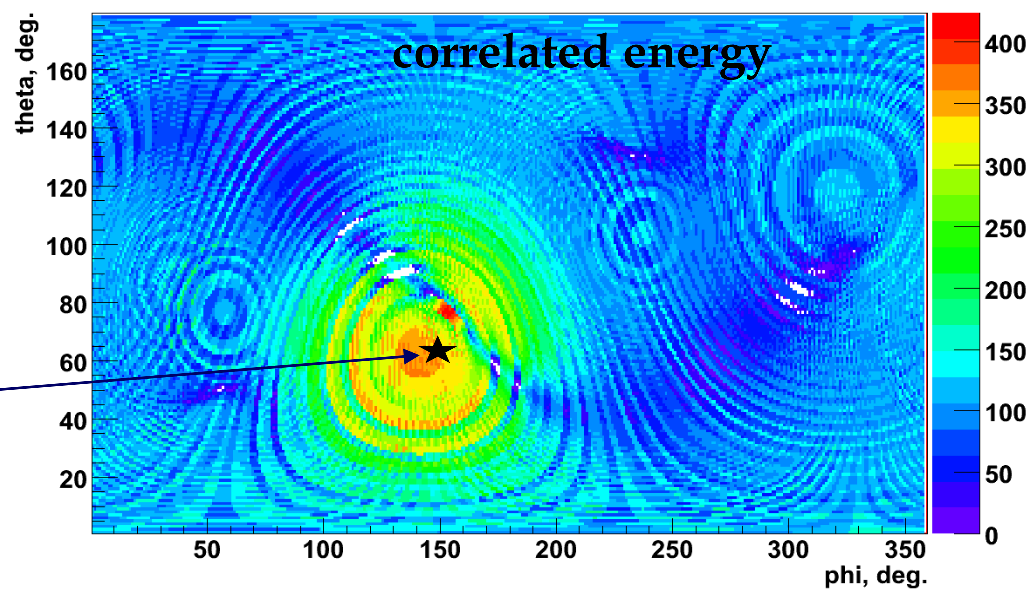
simulated sine-Gaussian waveform: f=1304, q=9,

	L1/H1/H2/G1
hrss(10^{-21}):	2.5/1.3/1.3/1.6
SNR(a) :	24 /16 / 8 / 5
F+ :	.25/.13/.13/.16

real noise, average amplitude SNR=14 per detector



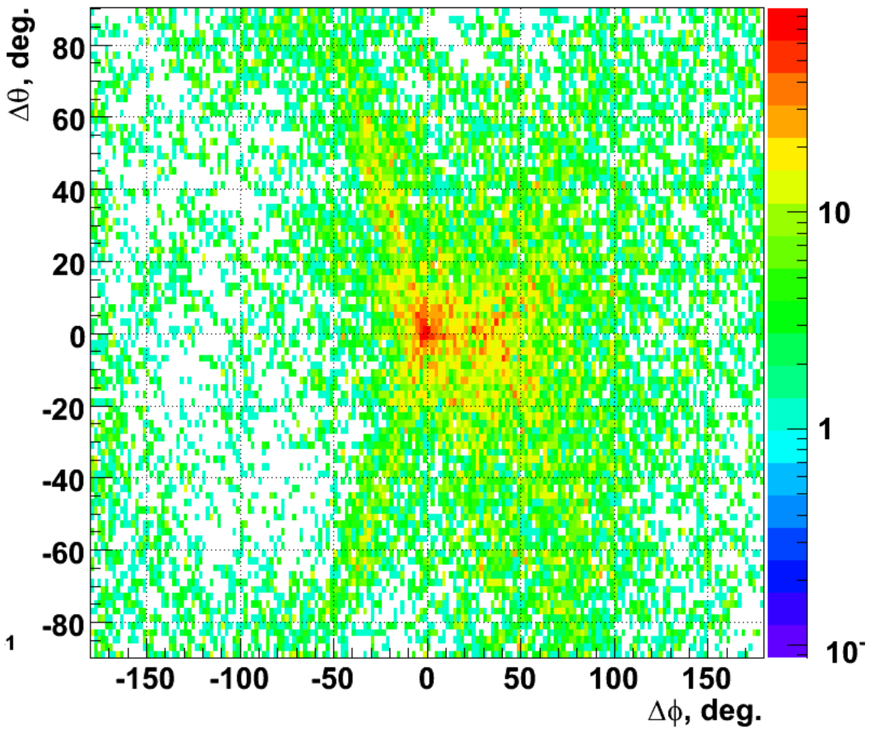
injection



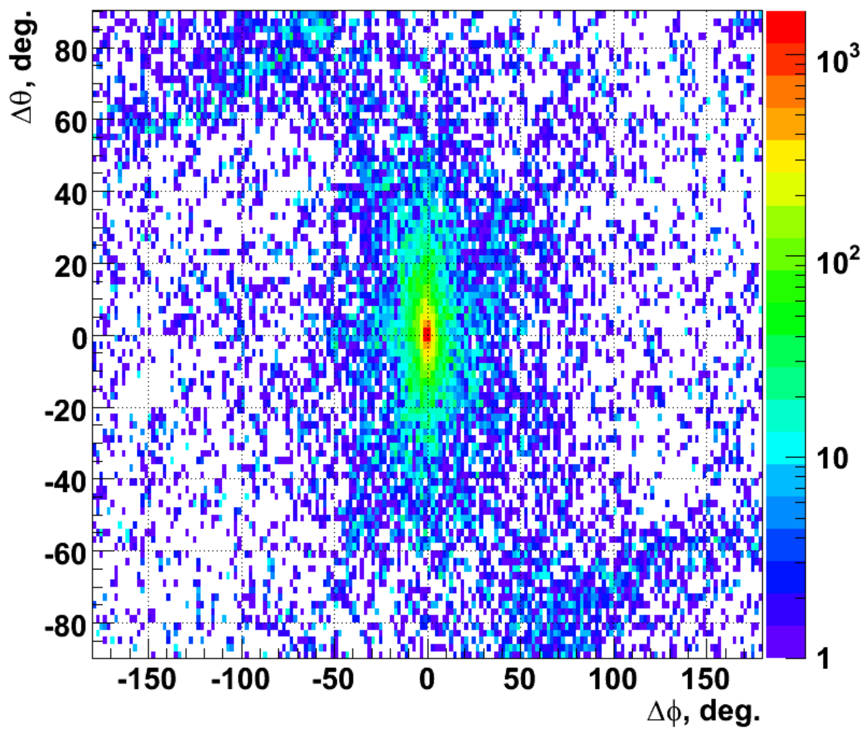
Coordinate reconstruction

S5 data

LIGO



LIGO+Virgo



Reconstruction of burst waveforms

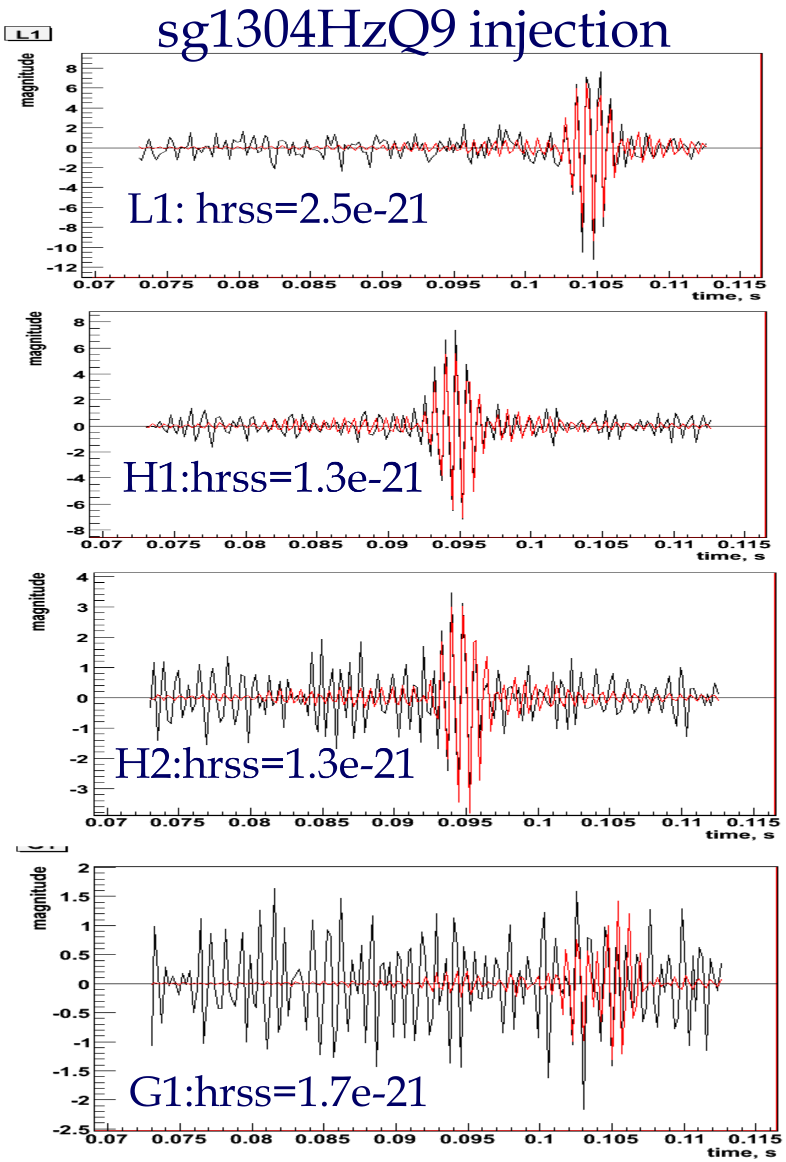
black band-limited time series

red reconstructed response

- If GW signal is detected, two polarizations and detector responses can be reconstructed and confronted with source models for extraction of the source parameters

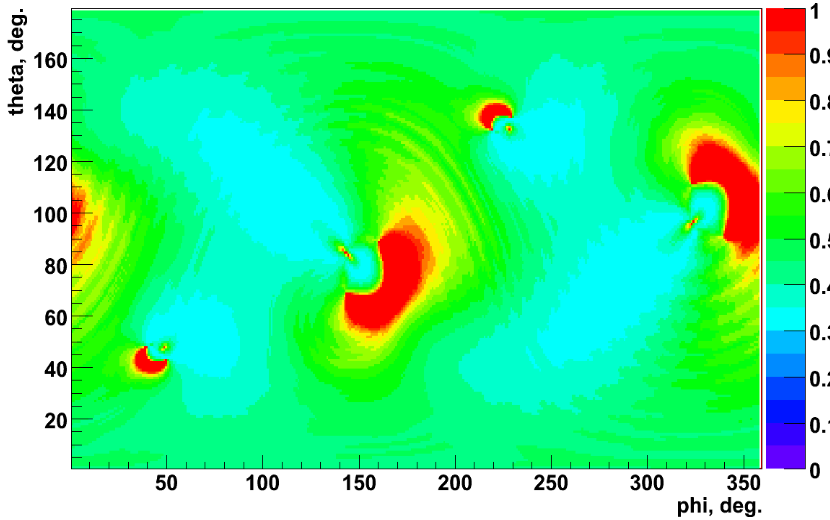
- Figures show an example of LIGO glitch reconstructed with the coherent WaveBurst event display (A.Mercer et al.)

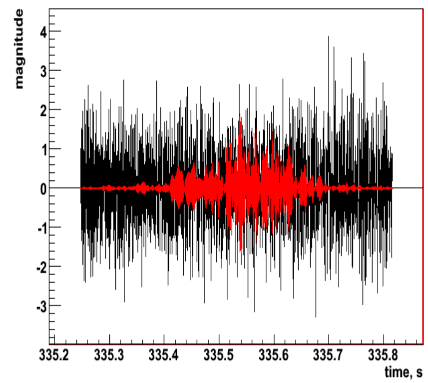
  → powerful tool for consistency test of coherent triggers.

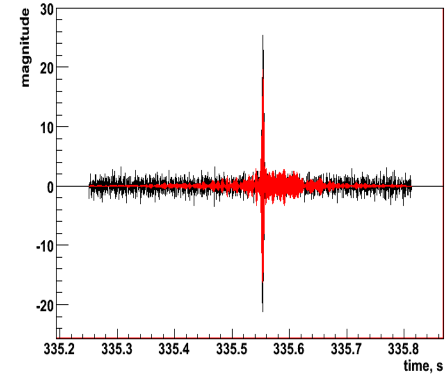


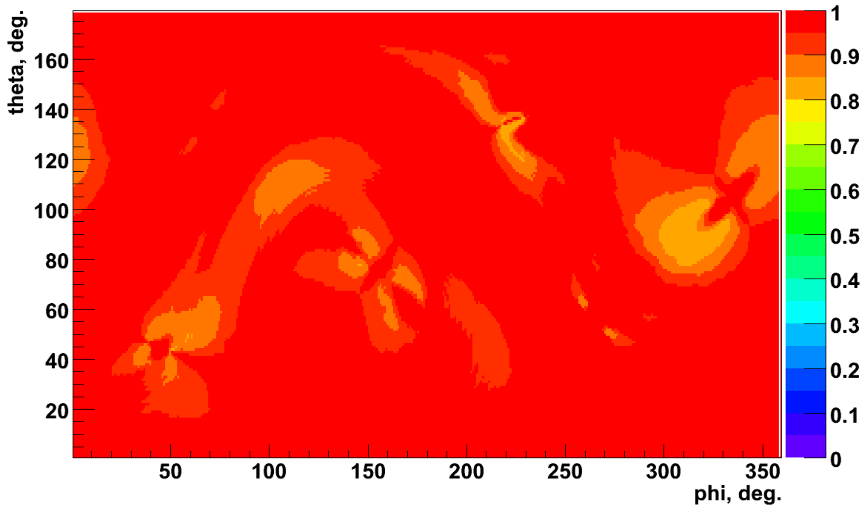
Likelihood penalty factor

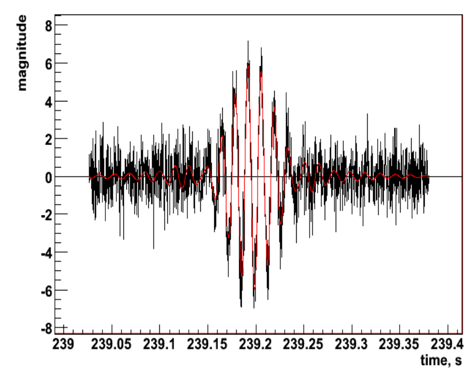
$$g = \frac{1}{\sqrt{1 - \min_k \left(\left\langle x_k^2 \right\rangle - \left\langle \xi_k^2 \right\rangle \right)}}$$

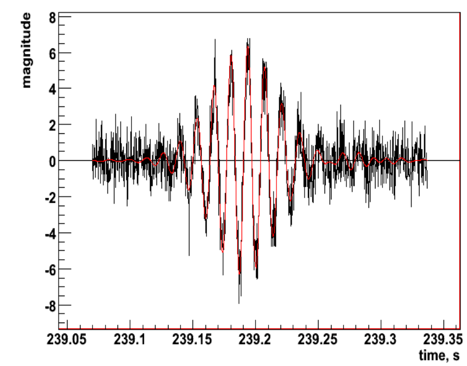












Likelihood constraints

- **Model independent constraint which requires that reconstructed responses ξ_k are orthogonal to reconstructed detector noise**

$$L = L_o(x,h) + \sum \lambda_k \left(\left\langle x_k \xi_k \right\rangle - \left\langle \xi_k^2 \right\rangle \right), \quad \xi_k = e_{+k} h_+ + e_{\times k} h_\times$$

- **If $\lambda_k = \lambda$ the constraint provides normalization of L over the sky in the presence of a regulator.**

$$h_+' = \alpha \left(\vec{X} \cdot \vec{e}_+' \right), \quad \alpha = \frac{\left\langle \left(\vec{X} \cdot \vec{e}_+' \right)^2 \right\rangle}{\left\langle \sum_k e_{+k}'^2 \left(\vec{X} \cdot \vec{e}_+' \right)^2 \right\rangle}$$

$\alpha(\theta,\phi)$ –likelihood normalization

- **Model dependent constraints can be used in the analysis → reduce signal parameter space and thus increase the detection efficiency**

Summary

- **Several GW detectors are now operating around the world forming a network**
- **Coherent network analysis addresses problems of detection and reconstruction of GW signals with detector networks**
- **Likelihood methods provide a universal framework for burst searches with arbitrary networks of GW detectors**
  - **matched filter for bursts**
  - **likelihood ratio statistic is used for detection**
  - **GW waveforms can be reconstructed from the data**
  - **location of sources in the sky can be measured**
  - **consistency test of events in different detectors**
- **Constraints significantly improve the performance of coherent algorithms**
- **Coherent algorithms are started to be used for burst searches**