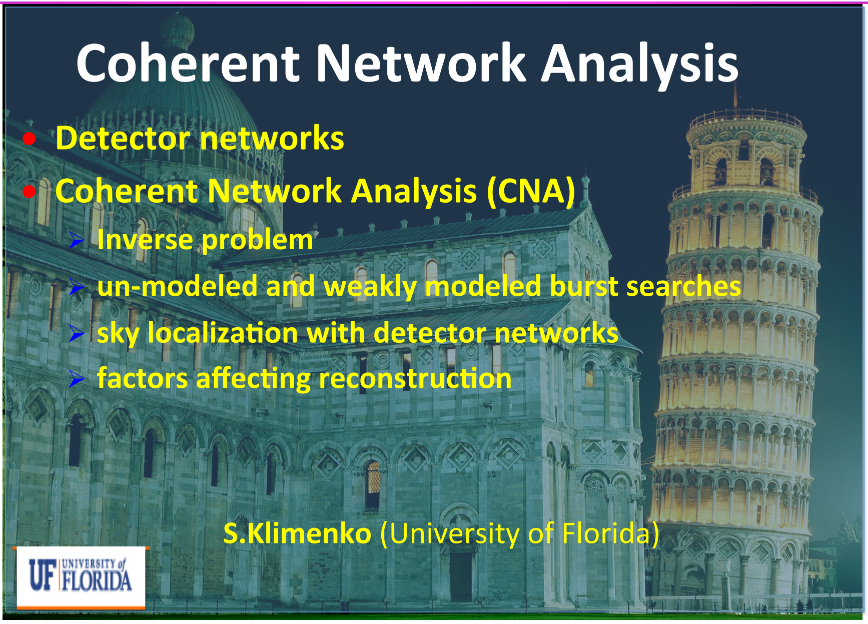

Coherent Network Analysis

- Detector networks
- Coherent Network Analysis (CNA)
  - Inverse problem
  - un-modeled and weakly modeled burst searches
  - sky localization with detector networks
  - factors affecting reconstruction

S.Klimenko (University of Florida)

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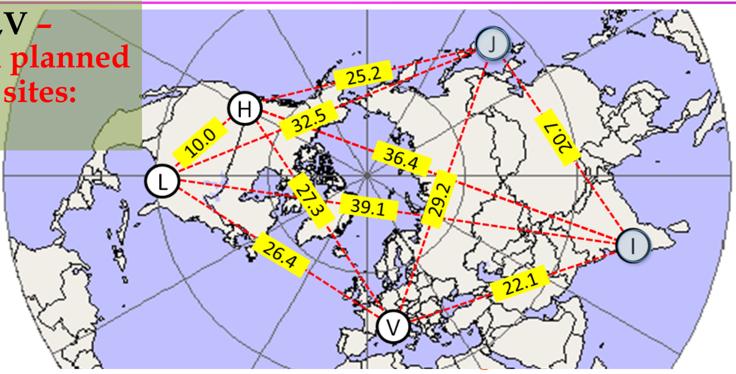
advanced (2G) detector network

I,J,H,L,V – **existing and planned detector sites:**



- aLIGO (H,L), aVirgo(V), KAGRA(J), LIGO-India (I), GEO HF (G)
  - target detection of anticipated NS-NS and possibly other sources after 2015.
- Coherent Network Analysis
  - detection and reconstruction of astrophysical GW sources with the world-wide network of GW detectors.

Objectives of Coherent Network Analysis

- Understand benefits and shortcomings of detector networks to detect sources and optimally capture science.
- Combine measurements from several detectors
  - ➢ elimination of instrumental/environmental artifacts
  - ➢ confident detection
  - ➢ reconstruction of source coordinates
  - ➢ reconstruction of GW waveforms
- CNA is a unified approach to handle
  - ➢ arbitrary number of detectors at different locations and arm's orientations
  - ➢ variability of detector responses as function of source coordinates
  - ➢ differences in the strain sensitivity of detectors
- Extraction of source parameters
  - ➢ confront measured waveforms with source models or include models

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Detector response to a GW signal

- **Antenna patterns**

$$F = \begin{bmatrix} \cos(2\psi) & \sin(2\psi) \\ -\sin(2\psi) & \cos(2\psi) \end{bmatrix} \begin{bmatrix} F_+[\theta,\phi] \\ F_\times[\theta,\phi] \end{bmatrix}$$

$$F_+[\theta,\phi] = \tfrac{1}{2}(1+\cos^2\theta)\cos 2\varphi$$

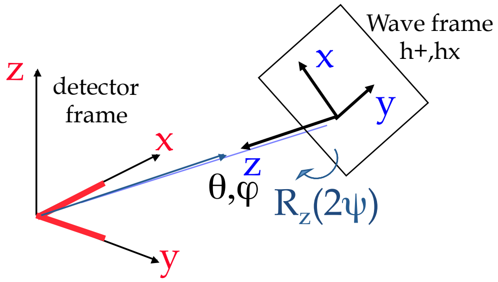
$$F_\times[\theta,\phi] = \cos\theta \sin 2\varphi$$

- **Sampled GW signal**

$$h[i] = \begin{bmatrix} \cos(2\psi) & \sin(2\psi) \\ -\sin(2\psi) & \cos(2\psi) \end{bmatrix} \begin{bmatrix} h_+[i] \\ h_\times[i] \end{bmatrix}$$

- **Sampled detector response**

$$\xi[i] = F_+ h_+[i] + F_\times h_\times[i] = F^T \cdot h[i]$$



- **Direction to the source θ,φ and polarization angle Ψ define relative orientation of the detector and wave frames.**
- **Rotation of the wave frame $R_z(2\Psi)$ induces transformations both for F and h, but ξ is INVARIANT**

In the analysis we have freedom to select any Ψ we like.

Whitened Network Response

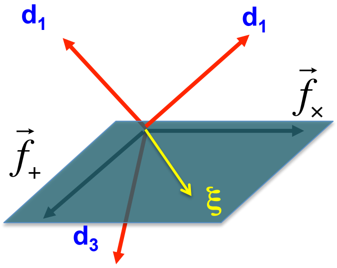
$$\vec{\xi}[i] = [\vec{f}_+^T[i], \vec{f}_\times^T[i]] \begin{bmatrix} h_+[i] \\ h_\times[i] \end{bmatrix} = f[i] \cdot \mathrm{h[i]}$$

- **Noise scaled network antenna patterns**
  - **in general time-frequency dependent**
  - **calculated for each TF data sample i characterized by noise PSD estimator S[i]**

$$\vec{f}_+[i] = \frac{F_{1+}(\theta,\phi,\psi)}{\sqrt{S_1[i]}},\ldots,\frac{F_{K+}(\theta,\phi,\psi)}{\sqrt{S_K[i]}},$$

$$\vec{f}_\times[i] = \frac{F_{1\times}(\theta,\phi,\psi)}{\sqrt{S_1[i]}},\ldots,\frac{F_{K\times}(\theta,\phi,\psi)}{\sqrt{S_K[i]}}$$

network plane



$$\vec{\xi} = (\xi_1,\ldots,\xi_K)$$

- **Dominant polarization wave frame:**

$$\vec{f}_+(\psi) \cdot \vec{f}_\times(\psi) = 0 \qquad |\vec{f}_+(\psi)| \geq |\vec{f}_\times(\psi)|$$

Network response to a GW event

- Consider a network event consisting of I TF samples

$$\begin{bmatrix} \xi[1] \\ \xi[2] \\ \ldots \\ \xi[I] \end{bmatrix} = \begin{bmatrix} f[1] & 0 & .. & 0 \\ 0 & f[2] & .. & 0 \\ .. & .. & .. & .. \\ 0 & 0 & .. & f[I] \end{bmatrix} \begin{bmatrix} h[1] \\ h[2] \\ \ldots \\ h[I] \end{bmatrix}$$

$$\Xi = \mathrm{F\ H}$$

  - Ξ – network response to a GW event
  - F – event network matrix
  - H – GW event amplitudes

- Network data stream X

$$\mathrm{X = F\ H + N}$$

  - N - network noise

Network SNR

$$\rho_{net} = 2\sqrt{\sum_k \int_0^{\infty} \left[\left| \vec{f}_{+k} h_+(f) \right|^2 + \left| f_{\times k} h_\times(f) \right|^2 \right] df}$$

- **GW rss amplitude** $h_{rss} = \sqrt{\int [h_+^2(t) + h_\times^2(t)]\,dt}$
- **Network noise** $S_{net} = \left(\sum_k S_k^{-1}\right)^{-1} \sim \frac{S_{\det}}{K}$
- **Population average SNR**
  - **assume** $h_{+rss} = h_{\times rss}$
  - **assume S(f_o)=const around characteristic signal frequency f_o**

$$\bar{\rho}_{net} \approx F \sqrt{\int_0^{\infty} \frac{|h_+(f)|^2 + |h_\times(f)|^2}{2 S_{net}(f)} df}$$

$$\bar{\rho}_{net} \approx \frac{F \cdot h_{rss}}{\sqrt{2 S_{net}}}$$

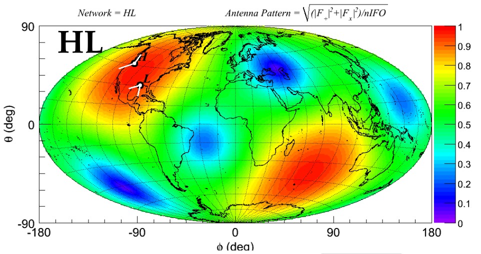
Schutz, *CQG* **28** 125023(2011)
Klimenko, et al PRD 83, 102001 (2011)

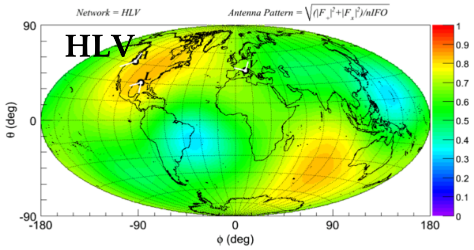
Network Acceptance

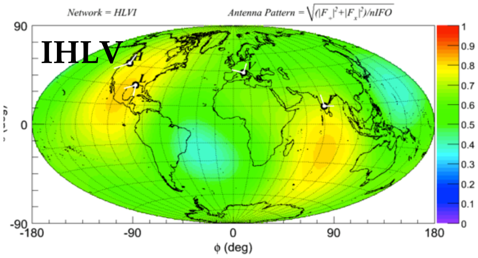
$$F(\omega) = \sqrt{\left(|f_+|^2 + |f_\times|^2\right) S_{net}}$$

Antenna sensitivity w.r.t
a network of
omni-directional detectors
(100% acceptance)

here and later for calculation of antenna patterns we assume equal sensitivity of all detectors – actual patterns are frequency dependent





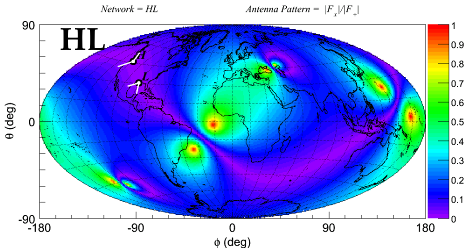


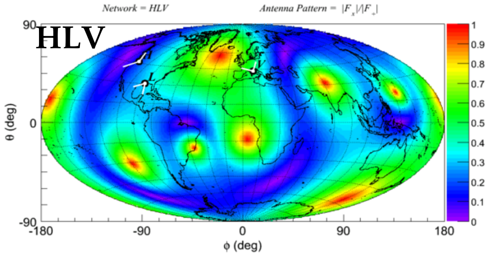
Network Alignment

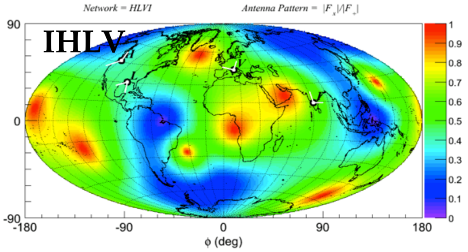
$$A = \frac{|f_\times|}{|f_+|}$$

- tells how well the wave polarization state is captured by the network

- for perfectly co-aligned detectors A=0 – detect only one GW component
- A - contribution to total network SNR from the second component







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cWB2G reconstruction stage

Event Trigger Generator

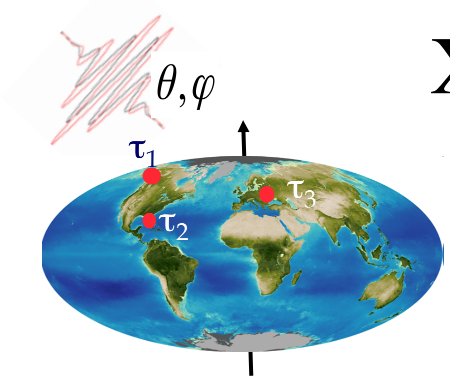
- ✓ Likelihood analysis
- ✓ un-modeled constraints
- ✓ modeled constraints
- ✓ cWB detection statistics
- ✓ sky localization
- ✓ parameter estimation

post-processing
- Selection cuts
- DQ & veto

cWB Reconstruction

- All-sky or limited sky
- Un-modeled/modeled searches
- Monster cluster analysis
- Sky localization statistics
- Waveform reconstruction
- Regulators
- Selection cuts
- Reconstruction of GW polarizations

Inverse Problem for GW transients



$$X = F \times H + N$$

data = network x wave + noise

Data analysis questions:

1. Detection: Is GW signal present in X?
2. Reconstruction: What can we learn about H from X?

DA scenarios:	known	unknown
● arrival time τ	ExtTrig	all-time
● arrival direction (θ,ϕ)	ExtTrig	all-sky
● GW waveforms	template	unmodeled

Search Method

- **search method – matched filter:**
  - **challenges: construct template bank ξ & search trough it**

data template

$$C(t\,|\,\Omega) = 4\int_0^\infty \frac{\tilde{x}(f)\;\tilde{\xi}^*(f\,|\,\Omega)}{S_n(f)}\, e^{2\pi i f t}\, df$$

detector noise

- **Two distinct MF approaches:**
  - **inspiral: construct accurate banks to accommodate for source parameter space Ω**
    - ✓ **modeled: Ω is defined by accurate astrophysical model of the source**
  - **burst: construct analytical banks of ad-hoc templates to accommodate for our ignorance of the source**
    - ✓ **un-modeled: Ω is defined by excess power in the data above detector noise**

Likelihood Method

Guersel&Tinto, 1989
lanagan & Hughes, 1998

- **Likelihood ratio** (global fit to GW data):
- **Noise model:** usually multivariate Gaussian noise

$$\Lambda = \frac{p(X\,|\,h)}{p(X\,|\,0)}$$

- **signal model (defined by detector response)**

$$p(X\,|\,0) \propto \exp[-X\Sigma^{-1}X^T] \qquad \Sigma\text{-noise covariance matrix}$$

$$\vec{\xi}[i] = h_+[i]\vec{F}_+ + h_\times[i]\vec{F}_\times, \quad h_+(\Omega), h_\times(\Omega), \quad \Omega - \text{signal model}$$

$$p(X\,|\,h) \propto \exp[-\left(X-\xi\right)\Sigma^{-1}\left(X-\xi\right)^T]$$

$$L = 2\ln\Lambda = 2\sum_i\left(\vec{X}[i]\cdot\vec{\xi}[i,h]\right) - \sum_i\left(\vec{\xi}[i,h]\cdot\vec{\xi}[i,h]\right)$$

- **find GW polarizations (h_+,h_x) at maximum of Λ**
- **find source sky location by variation of Λ over θ and ϕ**
- **Ambiguity due to a large number of free parameters**

Inspiral vs Bursts

$$\xi(\Omega,i) = F_+h_+[i] + F_\times h_x[i]$$

modeled(Inspiral)

- **ξ is calculated from theoretical waveforms h_+h_x described by source parameters: m1,m2**
- **Parameter space Ω is constrained by the model**
- **Sample Ω with templates (explicit template banks)**
- **Find τ, θ, ϕ, Ω (thus ξ) from best matching template**
- **Increase Ω by expanding models: spin, eccentricity, etc**

un-modeled(burst)

- **Amplitudes $h_+[i]$, $h_x[i]$ are free source parameters**
- **Parameter space is constrained by signal duration and bandwidth**
- **Search through parameter space analytically.**
- **Find τ, θ, ϕ, ξ from best matching template**
- **Decrease parameter space by adding astrophysical constraints**

conceptually the same method, but approaches are radically different

Standard likelihood solution for inspirals

"forward" approach

- Select source model
  - for example, non-spinning, non-eccentric BHs
- Select parameter space
  - range of total masses
  - range of mass ratios
  - … other parameters for more complex models
- Construct template bank of detector responses covering the source parameter space, inclination angles and sky locations. Make sure there are no cracks in the coverage – overlap > 0.98 between nearby templates
- Find matching template (and thus source parameters) at max likelihood
  - Find nearby templates to estimate errors

Standard likelihood solution for bursts

"inverse" approach

$$F = \begin{bmatrix} f[1] & 0 & .. & 0 \\ 0 & f[2] & .. & 0 \\ .. & .. & .. & .. \\ 0 & 0 & .. & f[I] \end{bmatrix}$$

- Select sky location (θ,ϕ)
  - calculate network matrix *F* for TF "event" {1,..,I}
  - Calculate data vector *X* by time-shifting data streams to synchronize detectors: $X = \{\vec{\mathrm{x}}[1],...,\vec{\mathrm{x}}[\mathrm{I}]\}$
- Parameterize GW signal: $H = \{\vec{h}[1],...,\vec{h}[[\mathrm{I}]\},\ h[i] = \left(h_{+}[i], h_{\times}[i]\right)$
- Find likelihood and its derivatives

$$L = 2\ln\Lambda = X^T(FH) + (FH)^T X - (FH)^T(FH) \qquad \frac{\partial L}{\partial h} = 0$$

- Solution for H is coherent combination of X
- Repeat for all-sky locations maximizing $L(H_s)$
- Find waveforms H_m and (θ_m,ϕ_m) at max{L}
- Confront waveforms with source models

$$H_s = \underbrace{\left(F^T F\right)^{-1} F^T}_{\text{Moore-Penrose inverse}} X$$

does not work for practical networks – MP inverse may not exist

Rank Deficiency of Network Matrix

$$F=\begin{bmatrix} f[1] & 0 & .. & 0 \\ 0 & f[2] & .. & 0 \\ .. & .. & .. & .. \\ 0 & 0 & .. & f[I] \end{bmatrix} \qquad \vec{\xi}[i]=[\vec{f}_+^T[i],\vec{f}_\times^T[i]]\begin{bmatrix} h_+[i] \\ h_\times[i] \end{bmatrix}=f[i]\cdot\mathrm{h[i]}$$

i – is a single sample of network response

- Multiply data $\vec{x}=\vec{\xi}+\vec{n}$ by the network pattern vectors (i is omitted)
  - DPF is assumed $(\vec{f}_+\cdot\vec{f}_\times)=0$ - diagonalize network matrix

$$\begin{matrix}(\vec{\xi}+\vec{n})\cdot\vec{f}_+ \\ (\vec{\xi}+\vec{n})\cdot\vec{f}_\times\end{matrix} \longrightarrow \underbrace{\begin{bmatrix}\vec{x}\cdot\vec{f}_+ \\ \vec{x}\cdot\vec{f}_\times\end{bmatrix}=\begin{bmatrix}|\vec{f}_+|^2 & 0 \\ 0 & |\vec{f}_\times|^2\end{bmatrix}\begin{bmatrix}h_+ \\ h_\times\end{bmatrix}}_{\partial L/\partial h_+=0,\ \partial L/\partial h_\times=0}+\begin{bmatrix}\vec{n}\cdot\vec{f}_+ \\ \vec{n}\cdot\vec{f}_\times\end{bmatrix}$$

- $|f_\times|<<|f_+|$ (A<<1) - hx can not be reconstructed from noisy data
- **need regulators – un-modeled constraints**

Klimenko, et al (2005)
Rakhmanov (2006)

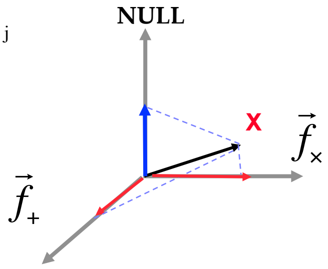
Network projections

- **To find statistic L_{max} we do not need explicit h_+ & h_x**
- **$L_{max} = L_+ + L_x$**

$$\begin{bmatrix}\vec{x}\cdot\vec{f}_+ \\ \vec{x}\cdot\vec{f}_\times\end{bmatrix}=\begin{bmatrix}|\vec{f}_+|^2 & 0 \\ 0 & |\vec{f}_\times|^2\end{bmatrix}\begin{bmatrix}h_+ \\ h_\times\end{bmatrix}$$

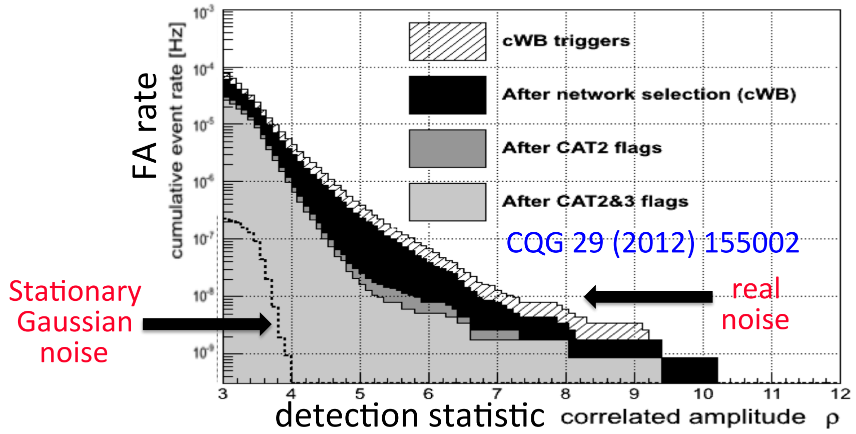
$$L_+=\frac{(\vec{x}\cdot\vec{f}_+)^2}{|\vec{f}_+|^2}=X^TP_+X, \quad P_{+ij}=\frac{f_{+i}f_{+j}}{|f_+|^2}=\mathrm{e}_{+\mathrm{i}}\mathrm{e}_{+\mathrm{j}}$$

$$L_+=\frac{(\vec{x}\cdot\vec{f}_\times)^2}{|\vec{f}_\times|^2}=X^TP_\times X, \quad P_{\times ij}=\frac{f_{\times i}f_{\times j}}{|f_\times|^2}=\mathrm{e}_{\times\mathrm{i}}\mathrm{e}_{\times\mathrm{j}}$$



- **L_{max} is never used as a detection statistic**

Real-life Detection



- Data is non-stationary, non-gaussian and affected by artifacts
- Empirical background sample for estimation of FA probability
  - constructed by time-shifting data → may be biased wrt true background
  - need a massive background set (T observation x 10^6)

Coherent Statistics

- True GW signal should be in the f_+, f_x plane

$$L = E - N$$

detected (signal) energy

total energy

noise (null) energy

- Likelihood quadratic form

$$L_{\max} = X^T P\ X, \quad P_{nm} = e_{+n}e_{+m} + e_{\times n}e_{\times m}$$

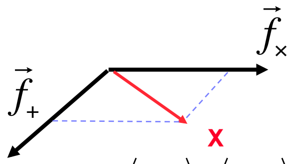
$$L = \sum_i \sum_{n,m} x_n[i]x_m[i]P_{nm}[i] = L_{i=j} + L_{i\neq j}$$

L matrix incoherent coherent

- Detection statistics
  - event ranking: characterize event strength, preferable if ~SNR
  - event consistency: significant null stream can be indication of a noise artifact

Two detector case

- no null space (any event is admitted as GW!)
- A << 1 for significant fraction of the sky
- L=const(θ,ϕ)



$$\xi_1 = x_1, \quad \xi_2 = x_2 \qquad L_+ + L_\times = \langle x_1 x_1 \rangle + \langle x_2 x_2 \rangle$$

- **Two detector paradox** (Mohanty et al, CQG 21 S1831 (2004))
  - no x-correlation term in the likelihood matrix! $L_{n \neq m} = 0!$
  - contradict to the case of two co-aligned detectors where

$$\xi_1 = \xi_2 = \frac{x_1 + x_2}{2}, \quad L_+ + L_\times = \tfrac{1}{2}\left[\langle x_1 x_1 \rangle + \langle x_2 x_2 \rangle + 2\langle x_1 x_2 \rangle\right]$$

power

cross-correlation

- What is the meaning of coherent energy?

In-coherent/Coherent Energy

$$L_+ = \sum_{i,j} x_i x_j P_{ij,+} = E_{+(i=j)} + C_{+(i \neq j)}$$

$$L_\times = \sum_{i,j} x_i x_j P_{ij,\times} = E_{\times(i=j)} + C_{\times(i \neq j)}$$

- quadratic forms C_+ & C_x depend on time delays between detectors and carry information about θ,ϕ – sensitive to source coordinates
- properties of the likelihood quadratic forms

arbitrary network

$$\mathrm{cov}(L_+ L_\times) = 0$$

$$\mathrm{cov}(C_+ C_\times) = -\sum \mathrm{e}_{+\mathrm{i}}^2 \mathrm{e}_{\times\mathrm{i}}^2$$

$$\mathrm{cov}(E_+ E_\times) = \sum \mathrm{e}_{+\mathrm{i}}^2 \mathrm{e}_{\times\mathrm{i}}^2$$

2 detector network

$$C_+ + C_\times = 0$$

$$E_+ + E_\times = \mathrm{x}_1^2 + \mathrm{x}_2^2$$

- E+, Ex, C+, Cx are dependent
- How should we calculate "generalized" network x-correlation?

THE Projection Operator

- **Construction of the projection operator**

$$P_{nm} = e_{+n}e_{+m} + e_{\times n}e_{\times m}$$

is ambiguous: $e_+e_+ \rightarrow rotation \rightarrow e_+^{'}e_+^{'}$

$$L_{\max} = X^T P\ X = X^T P'\ X$$

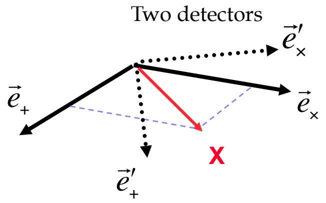
- **incoherent & coherent terms are not invariant**
- **Select the projection operator as**

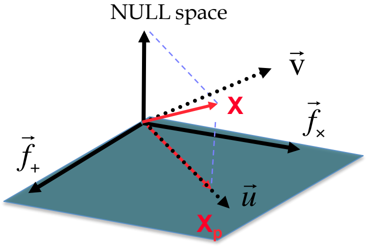
$$P_{nm} = u_n u_m$$

(*solves two-detector paradox*)

- **coherent/incoherent energies**

$C\ = X^T P_u(n \neq m)X$ $\qquad E_I = X^T P_u(n = m)X$





$$\vec{u} \cdot \vec{\mathrm{v}} = 0$$

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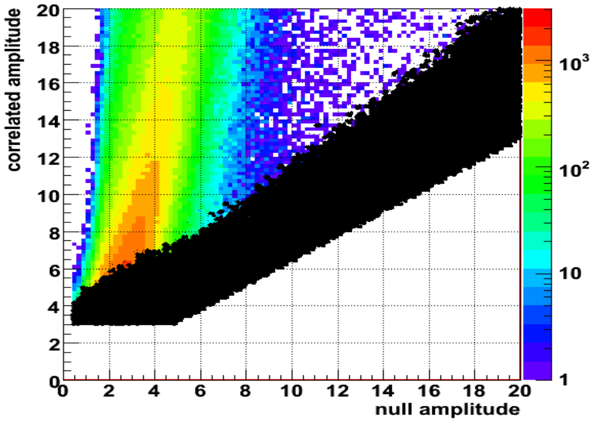
coherent – null energy

- **coherent energy: sum of the off-diagonal elements of L matrix**

$$E_{coherent} = \sum\nolimits_{i \neq j} L_{ij}$$

- **null energy *null*: energy of the reconstructed detector noise**

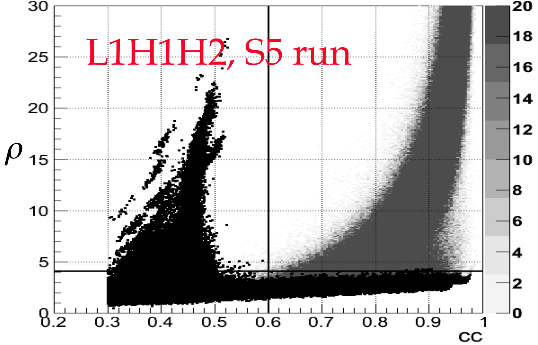
$$\sqrt{E_{coherent}}$$



Rejection of glitches

- Coherent statistics
  - Network correlation coefficient cc - rejection of glitches
  - network correlated amplitude η – event ranking statistic

$$cc = \frac{E_{i\neq j}}{N + E_{i\neq j}} \qquad \rho = \sqrt{\frac{cc \cdot E_{i\neq j}}{K}}$$



black - background
gray – GW injections

PRD 81 (2010) 102001

Use also DQ and Veto: characterization of detector noise is one of the most challenging tasks in the GW experiment

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cwb2G Coherent statistics

$$cc = \frac{E_{coh}}{N + E_{coh}} \qquad \rho_{1G} = \sqrt{\frac{E_{coh} \cdot cc}{K}}$$

- Rejection of glitches
  - Network correlation coefficient cc also used as sky localization
  - Sub-network consistency (new)

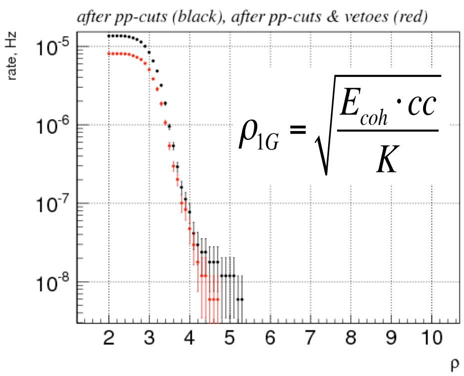
$$cc_{2G} = \frac{\langle L \cdot cc \rangle}{E}$$

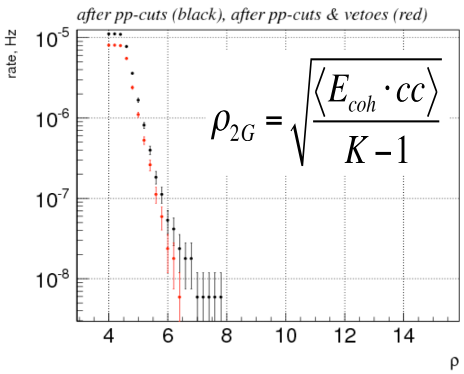
$$sc = \frac{E_{sub}}{N + E_{sub}}$$

- event ranking statistic
  - Network coherent amplitude ρ
  - ρ_{2G} has a meaning of coherent SNR
  - ρ_{2G}~ ρ_{MF} for similar responses in detectors

$$\rho_{2G} = \sqrt{\frac{\langle E_{coh} \cdot cc \rangle}{K-1}}$$

FAR

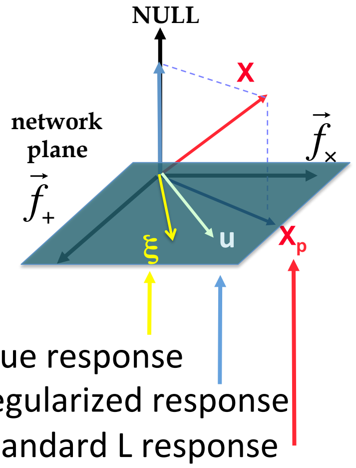




- Regulators are critical for FAR reduction
- FAR~10-9 for Gaussian noise: ρ_{1G}~4; ρ_{2G}~6

Network Regulators

- **For existing networks the standard projection is rarely an optimal solution**
- **Network regulators → construct P by guessing orientation of the projection vector u (Klimenko et al, 2005)**
  - **hard regulator: u is pointing along f_+ - gives optimal solution for closely aligned networks**
  - **Other regulator types can be constructed**
- **2G regulators**
  - **de-noising: Ec > gamma * Lx**
  - **network: |fx|^2*L > Ni*delta*Lx**



true response
regularized response
standard L response

Dual stream likelihood analysis

$$L = 2\left(\vec{x}\cdot\vec{\xi}\right) - \left(\vec{\xi}\cdot\vec{\xi}\right)$$

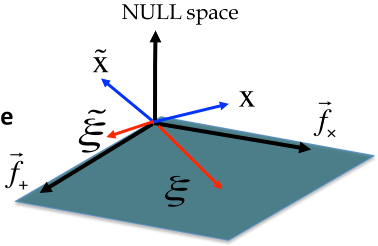
$$\tilde{L} = 2\left(\vec{\tilde{x}}\cdot\vec{\tilde{\xi}}\right) - \left(\vec{\tilde{\xi}}\cdot\vec{\tilde{\xi}}\right)$$

- **Dual data stream:** x and $\tilde{x}$ - quadrature
  - quadrature stream contains the same information as x
  - not true for a group of data samples (event)
- Better collection of energy

$$x = x'\cos(\lambda) + \tilde{x}'\sin(\lambda)$$

$$\tilde{x} = \tilde{x}'\cos(\lambda) - x'\sin(\lambda)$$

- Phase transformation
  - L+L-tilde is λ-invariant (but not individual Ls)
  - Apply phase transformation to x , $\tilde{x}$ to make their projections ξ , $\tilde{\xi}$ orthogonal to each other
- ξ - Principle Dual Stream Component



$$\vec{\xi}\cdot\vec{\tilde{\xi}} = 0$$

Reconstruction of GW polarizations

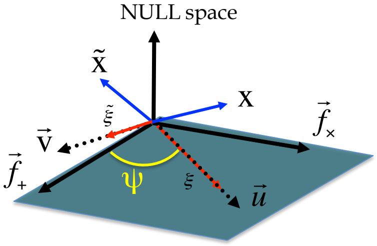
- Assuming DPF and applying dual stream phase transformation, GW responses are parameterized as

$$x = x'\cos(\lambda) + \tilde{x}'\sin(\lambda)$$

$$\tilde{x} = \tilde{x}'\cos(\lambda) - x'\sin(\lambda)$$

$$\xi = h_o\vec{u}(\psi)\frac{\cos\iota}{\cos\psi}|f_+|, \quad \tilde{\xi} = h_o\vec{v}(\psi)\frac{\sin\iota}{\cos\psi}|f_\times|$$

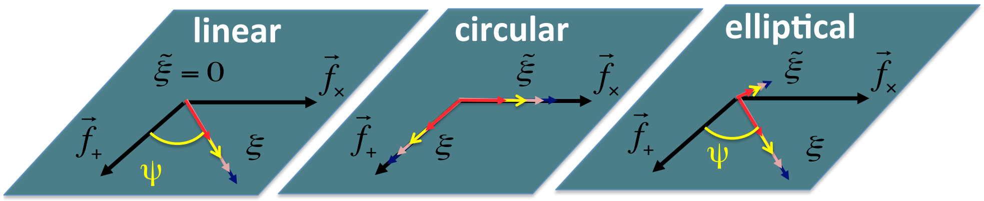
  - ι – instantaneous ellipticity angle
  - ψ – instantaneous polarization angle
  - ho - GW strain amplitude
- Wave polarization is captured as a pattern of ξ , $\tilde{\xi}$ vectors



$$\vec{u}\cdot\vec{v} = 0$$

Capturing GW polarizations

- **Signal polarization is defined by its $(\xi, \tilde{\xi})$ pattern**
  - **elliptical polarization is characterized by wave ellipticity** $|\tilde{\xi}| / |\xi|$
  - **no correlation with other source parameters**

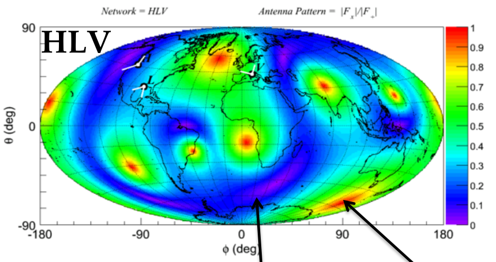


- **However, the unique polarization pattern may be distorted by**
  - **noise - results in the pattern dispersion**
  - **network – results in the pattern bias**

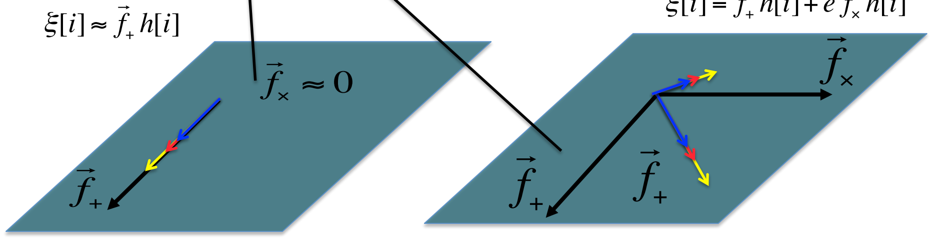
With sufficient network alignment coverage,

wave ellipticity and ψ measured from the pattern, are uniquely mapped to the source parameters (inclination and polarization angles)

Effect of network alignment



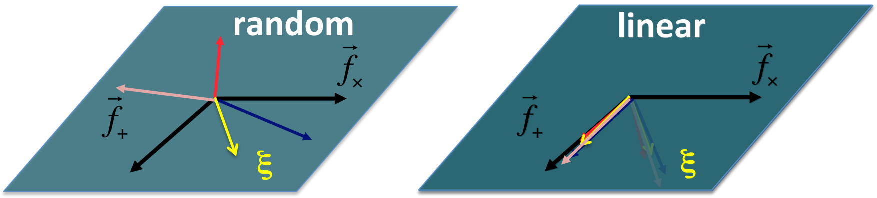
- Example: elliptical wave patterns
- For A~0 network can not distinguish polarization state of incoming wave



Full alignment coverage is important for reconstruction of gravitational wave polarizations

Reconstruction of GW polarizations

- **Signal polarizations can be reconstructed from the dual stream data $(w,\tilde{w})$ in the WDM domain**
- **Signal polarization is associated with a pattern of reconstructed responses $(\xi,\tilde{\xi})$**

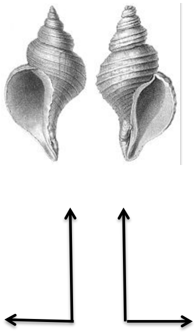


- **However, GW polarization h_x may not be recovered from noisy data when $|f_+| >> |f_\times|$**

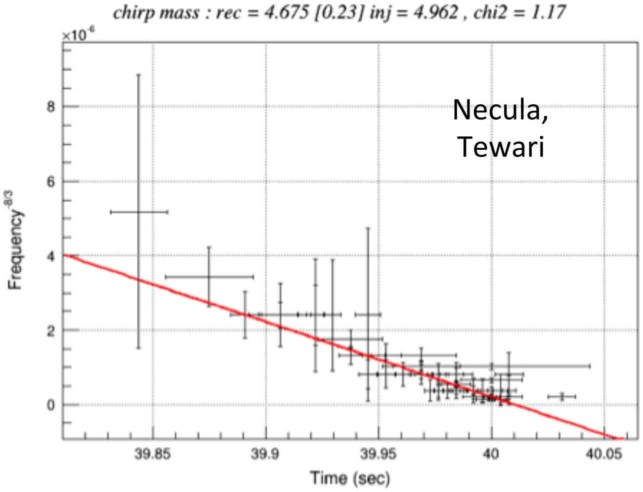
2G cWB searches

- ‘r’ - un-modeled
- ‘i’ - iota – wave (fixed chirality)
  - use for all-sky search instead of ‘r’-search
- ‘p’ - Psi - wave (*const* polarization angle)
- ‘l’,’s’ – linear, loose linear
- ‘c’,’g’ – circular, loose circular
  - use ‘g’ for inspiral (eBBH, IMBH) searches
- ‘e’ – elliptical

chirality



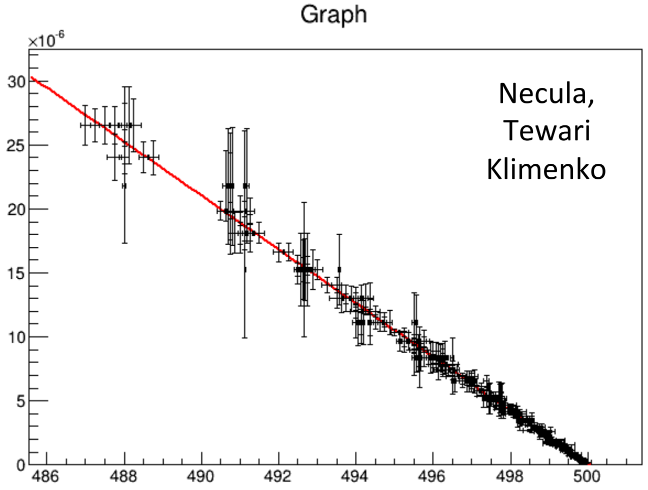
Big Dog Chirp Mass



chirp mass : rec = 4.675 [0.23] inj = 4.962 , chi2 = 1.17

- **Reconstruction of chirp mass from TF data.**
- **Can be used for background reduction**

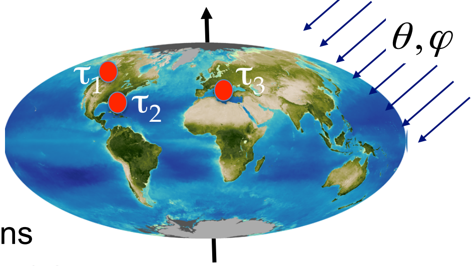
NS-NS in aLIGO noise



Graph

- **cWB2G can detect signals much longer than 1 sec**

Source Localization



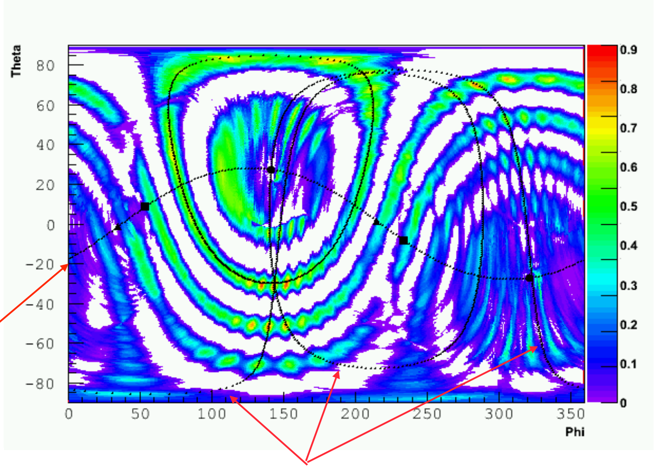
- **Two basic methods**
  - tangulation ($\tau_1,\tau_2,\tau_3,..$)
    - 3 or more sites
  - variability of antenna patterns
    - requires good coverage of GW polarizations

- **Coherent network analysis employs both of them. Sky location is determined at max L or C for both modeled (template) and un-modeled (burst) methods**
  - **modeled: more accurate, can be biased by model**
  - **un-modeled: less accurate, more robust**

Probability Sky Map

PSM shows how consistent are reconstructed waveforms and time delays as function of θ,ϕ. Source location is at PSM max.

probability map: coherent network analysis

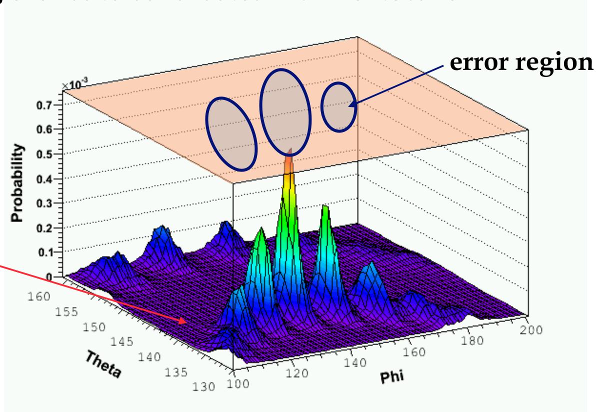


detector plane

constant delay rings for detector pairs

Error Regions

- Source location is characterized by a spot in the sky (error region) rather than by a single (θ,ϕ) direction
  - x% error region - a sky area with the cumulative probability of x%
- The coverage of error regions has to be validated with MonteCarlo



error region

Probability map

- Error regions can be reported for optical/radio followup →multi-messenger observations

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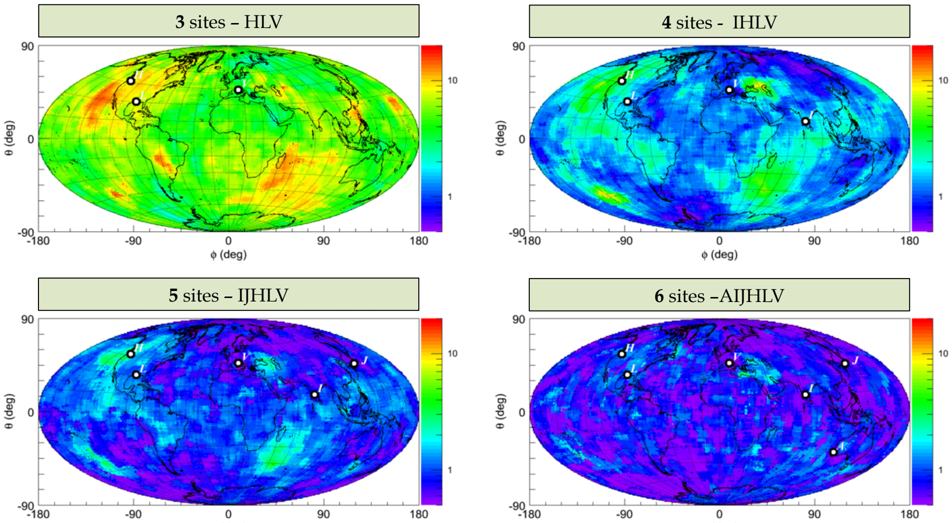
Sky Localization



3 sites – HLV

4 sites - IHLV

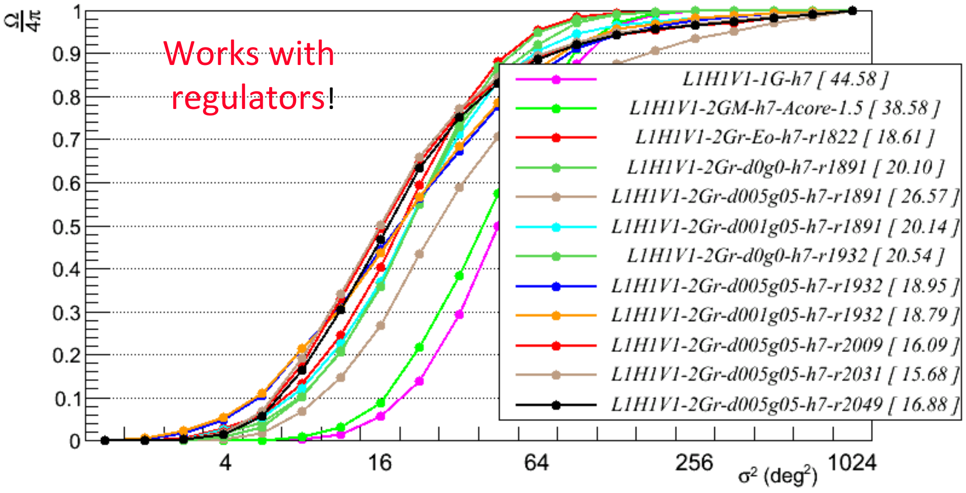
5 sites – IJHLV

6 sites –AIJHLV

- Median error angle (50% CL, SNRnet<30) for "worse case scenario" for reconstruction of ad-hoc signals in the bucket (200 Hz-300 Hz)
  - obvious observation – more sites is better

Sky Localization

Cumulative fraction of the sky as a function of the 50% error region



Works with regulators!

Reconstruction Summary

- read network event from trigger file
- calculated time-delayed amplitudes
- read WDM x-talk catalog (used in monster analysis)
- run sky-loop (find optimal sky location)
  - identify event TF amplitudes for each sky location (network pixels with E>Eth)
  - calculate standard coherent energy → dismiss sky location if too low
  - apply polarization constraint
  - apply network constraint
  - apply de-noising constraint
  - Calculate coherent statistics
- for optimal sky locations
  - get multi-resolution coherent statistics
  - do monster analysis → get corresponding coherent statistics
  - do chirp mass reconstruction
  - calculate sky error regions

References

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- Constraint likelihood: Klimenko et al, PRD 72, 122002 (2005)
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  - first introduction of likelihood constraints/regulators
- Penalized likelihood: Mohanty et al, CQG 23 4799 (2006).
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