A Wavelet-based tool for non-stationary PSDs

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Introduction

Wiener-Khinchin theorem states that a stationary covariance matrix,

$$C_{\rm s}(t,t') = \int_0^\infty df\, S(f)\cos(2\pi f(t-t')), \qquad (1)$$

where $S(f)$ is the power spectral density (PSD). The integral can be approximated as the sum

$$\begin{aligned}&\sum_a \Delta f\, S(f_a)\cos(2\pi f_a(t-t'))\\ &= \sum_a \Delta f\, S(f_a)\left[\cos(2\pi f_a t)\cos(2\pi f_a t')\right.\\ &\quad \left.+\sin(2\pi f_a t)\sin(2\pi f_a t')\right]\end{aligned} \qquad (2)$$

Problem : What if $S(f)$ is in fact really $S(f,t)$?
Solution : Use wavelets to track the time-evolution of $S(f,t)$.

Let $g_i(t)$ be the i^{th} envelope of a mother wavelet that satisfies the condition

$$\sum_i g_i(t) = 1. \qquad (3)$$

Then, in the case of a non-stationary C_{ns}, (2) can be expressed as,

$$\begin{aligned}C_{ns}(t,t') &= \sum_{a,i,j} \Delta f\, S(f_a)\left[g_i(t)\cos(2\pi f_a t) g_j(t')\cos(2\pi f_a t')\right.\\ &\qquad \left.+ g_i(t)\sin(2\pi f_a t) g_j(t')\sin(2\pi f_a t')\right]\\ &= \sum_{a,i,j} \Delta f\, S(f_a)\left[g_i(t) g_j(t')\cos(2\pi f_a (t-t'))\right] \end{aligned} \qquad (4)$$

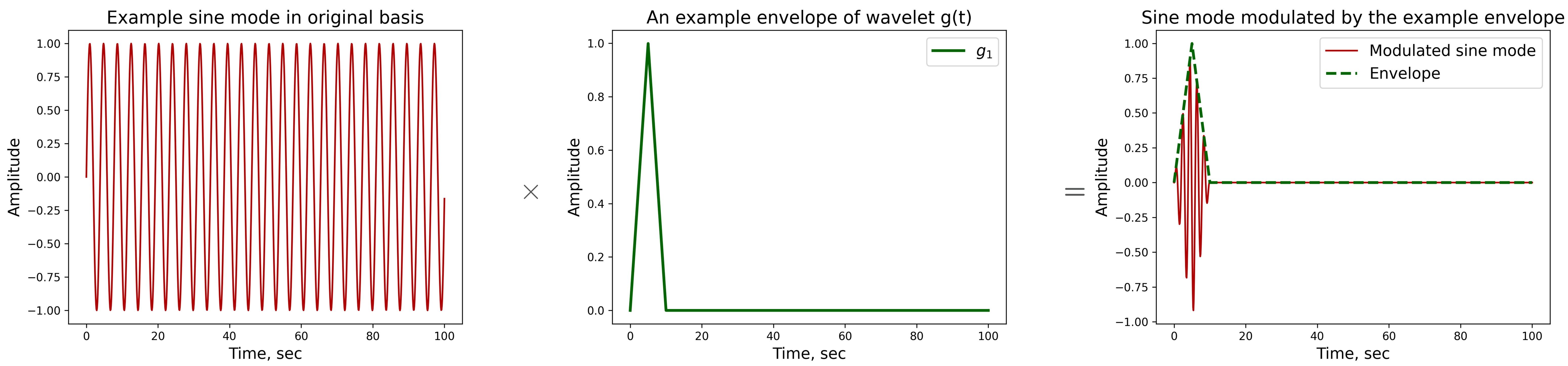
$$C_{\rm ns}(t,t') = g(t) C_{\rm s}(t-t') g(t') \qquad (5)$$

as also found by Falxa et.al [2].

☞ Fourier modes are being modulated by time evolving wavelets.

Method Demonstration

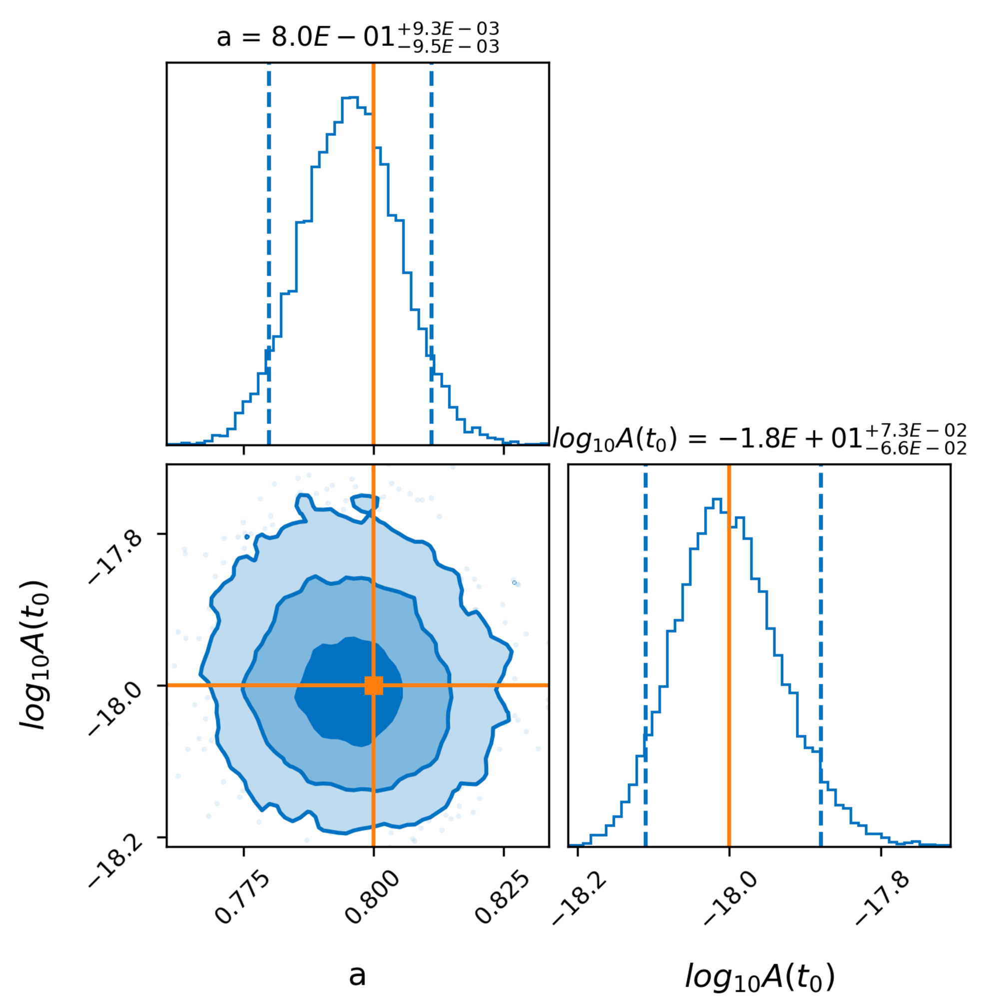
In a Low-Rank Approximation [1], $C = N + F\Phi F^T$, where, N contains measurement errors of TOAs, and F is the Fourier Design Matrix, $\Phi_{\mu\nu} = \delta_{\mu\nu} S(f_\mu)\Delta f$, the stationary PSD. In the Fourier-wavelet-basis, we choose n_w wavelets to modulate F and $\Phi_{\mu\nu t} = \delta_{\mu\nu} S(f_\mu, t)\Delta f$. As an example,



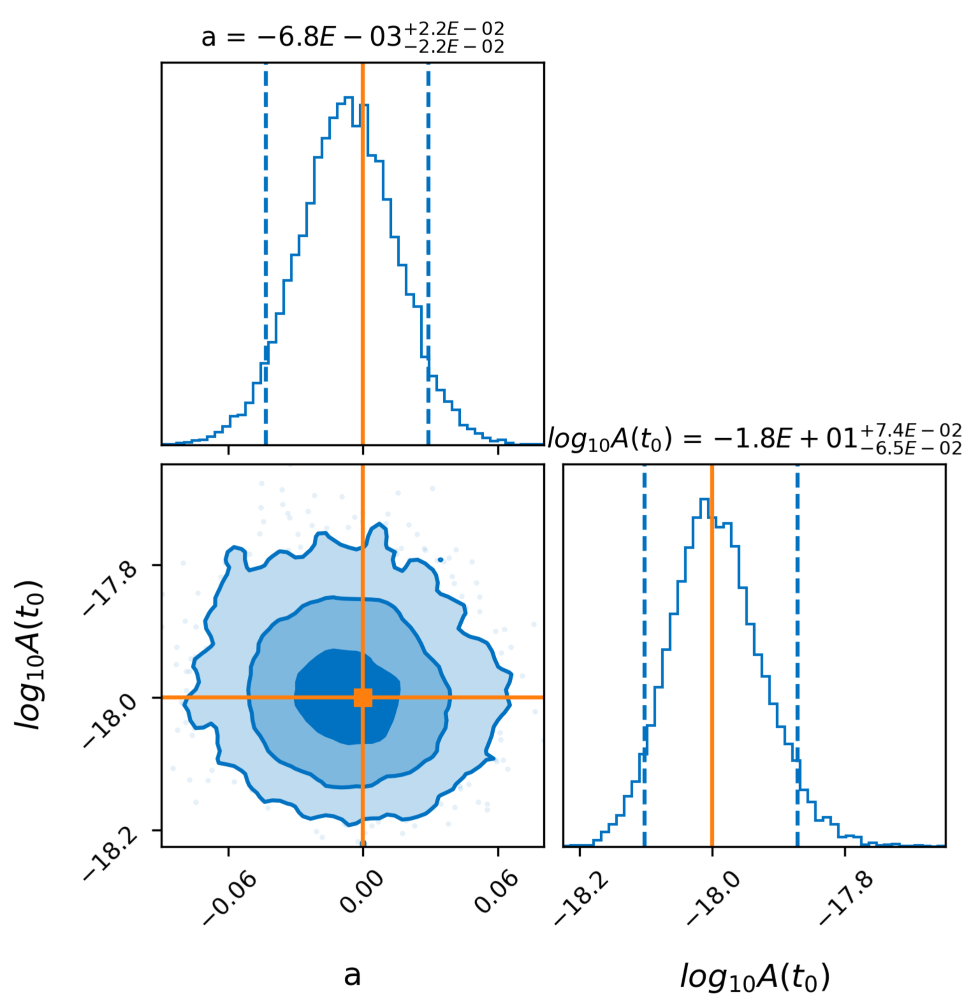
Results from simulations

For PSD Model : $S(f,t) = A^2(t)\left(\frac{f}{f_0}\right)^{-\gamma} {f_0}^{-3}$, $\log_{10} A(t) = \log_{10} A(t_0) + a\frac{(t-t_0)}{T}$

Case (i) Non-Stationary PSD: $A(t_0) = 10^{-18}$ and $a = 0.8$



Case (ii) Stationary PSD: $A(t_0) = 10^{-18}$ and $a = 0$



Future work

(i) Explore other mother wavelets (ii) Combine with the Fourier-domain likelihood (Talk by Serena Valtolina) (iii) Apply to real data

References

[1] van Haasteren & Vallisneri, MNRAS**446**, (2015) [2] Falxa et. al, PRD**109**, (2024) [3] Priestley, J.R.Stat.Soc.Ser **204** (1965)